

1. **(5 points)** Using the difference quotient, calculate the derivative of $f(x) = 2x - 5x^2$.

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{[2(x+h) - 5(x+h)^2] - (2x - 5x^2)}{h} \\ &= \lim_{h \rightarrow 0} \frac{2x + 2h - 5x^2 - 10xh - 5h^2 - 2x + 5x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{2h - 10xh - 5h^2}{h} = \lim_{h \rightarrow 0} 2 - 10x - 5h = 2 - 10x \end{aligned}$$

2. **(4 points)** Calculate $\frac{d^3}{dt^3} (3e^t - 2\sin t + 5t^3 - 2t)$.

Three differentiations will leave e^t unchanged, convert $\sin t$ to $-\cos t$, annihilate $2t$, and convert $5t^3$ to $5 \cdot 3 \cdot 2 \cdot 1t^0$, so the third derivative will be $3e^t + 2\cos t + 30$.

3. **(6 points)** If $g(u) = \frac{e^u + \sec u}{u^3 + \sqrt{u}}$, calculate $g'(u)$.

We will use the quotient rule to find that

$$\begin{aligned} g'(u) &= \frac{(u^3 + \sqrt{u}) \frac{d}{du}(e^u + \sec u) - (e^u + \sec u) \frac{d}{du}(u^3 + u^{1/2})}{(u^3 + \sqrt{u})^2} \\ &= \frac{(u^3 + \sqrt{u})(e^u + \sec u \tan u) - (e^u + \sec u) \frac{d}{du}(3u^2 + \frac{1}{2}u^{-1/2})}{(u^3 + \sqrt{u})^2} \end{aligned}$$

4. **(5 points)** For $y = x^3 \cot x$, compute $\frac{dy}{dx}$.

Using the product rule,

$$\frac{dy}{dx} = \left(\frac{d}{dx} x^3 \right) \cot x + x^3 \frac{d}{dx} \cot x = 3x^2 \cot x - x^3 \csc^2 x.$$

5. **(2 point bonus)** On the back of this page, find a general formula for $\frac{d^n}{dx^n} (e^x \sin x)$.