1. **(7 points)** *Given the equation $e^y = x + xy$, determine $\frac{dy}{dx}$ by implicit differentiation.*

   We differentiate each side with respect to x, using the product rule and chain rule as necessary:

$$\begin{aligned}
\frac{d}{dx}e^y &= \frac{d}{dx}(x+xy) \\
\frac{dy}{dx}\frac{d}{dy}e^y &= 1 + \left(\frac{d}{dx}x\right)y + x\left(\frac{d}{dx}y\right) \\
\frac{dy}{dx}e^y &= 1 + 1y + x\frac{dy}{dx}
\end{aligned}$$

   At this point the calculus is complete and all we need to do is algebraically isolate the $\frac{dy}{dx}$:

$$\begin{aligned}
\frac{dy}{dx}e^y &= 1 + y + x\frac{dy}{dx} \\
e^y\frac{dy}{dx} - x\frac{dy}{dx} &= 1 + y \\
(e^y - x)\frac{dy}{dx} &= 1 + y \\
\frac{dy}{dx} &= \frac{1+y}{e^y - x}
\end{aligned}$$

2. **(6 points)** *Find the derivative with respect to x of the expression $x\sqrt{1+e^x}$.*

   This is on the most straightforward level a product of two things, so we will use the product rule. However, one of those two factors, $\sqrt{1+e^x}$, is a composition so we will find ourselves using the chain rule. With that game plan in mind, we proceed to perform the differentiation:

$$\begin{aligned}
\frac{d}{dx}\left(x\sqrt{1+e^x}\right) &= \left(\frac{d}{dx}x\right)\sqrt{1+e^x} + x\frac{d}{dx}\sqrt{1+e^x} \\
&= 1\sqrt{1+e^x} + x\frac{d}{dx}\sqrt{u} \text{ for } u = 1+e^x \\
&= \sqrt{1+e^x} + x\frac{du}{dx}\frac{d}{du}u^{1/2} \\
&= \sqrt{1+e^x} + x \cdot e^x \cdot \frac{1}{2}u^{-1/2} \\
&= \sqrt{1+e^x} + x \cdot e^x \cdot \frac{1}{2}(1+e^x)^{-1/2} \\
&= \sqrt{1+e^x} + \frac{xe^x}{2\sqrt{1+e^x}}
\end{aligned}$$

   The last line is a cosmetic improvement and is unnecessary.

3. **(7 points)** *If $y = \tan(\frac{x^2+1}{x^3+1})$, calculate $\frac{dy}{dx}$.*

   On its outermost level this expression is a composition, but inside the composition there is a quotient, which will necessitate the quotient rule. We perform this calculation below.

$$
\begin{aligned}
\frac{dy}{dx} &= \frac{d}{dx}\tan(u) \text{ for } u = \frac{x^2+1}{x^3+1} \\
&= \frac{du}{dx}\frac{d}{du}\tan(u) \\
&= \left(\frac{d}{dx}\frac{x^2+1}{x^3+1}\right)\sec^2(u) \\
&= \frac{(x^3+1)\frac{d}{dx}(x^2+1) - (x^2+1)\frac{d}{dx}(x^3+1)}{(x^3+1)^2}\sec^2\left(\frac{x^2+1}{x^3+1}\right) \\
&= \frac{(x^3+1)(2x) - (x^2+1)(3x^2)}{(x^3+1)^2}\sec^2\left(\frac{x^2+1}{x^3+1}\right)
\end{aligned}
$$

4. **(2 point bonus)** *Presuming that the function f is invertible, find a formula on the back of this page for the derivative of the function f^{-1} in terms of the functions f, f', and f^{-1}.*