1. **(8 points)** *An industrial gravel crusher is pouring gravel out at a rate of 2 cubic meters per second, forming a conical pile whose height is twice its radius. If the pile is currently 10 meters high, how quickly is its height increasing? (Note that a cone of radius r and height h has volume $\frac{1}{3}\pi r^2 h$.)*

   Let r, h, and V represent the radius, height, and volume of the gravel pile respectively, and let t represent time. Parsing the descriptions above, we see that $\frac{dV}{dt} = 2$, $h = 2r$, and that $V = \frac{1}{3}\pi r^2 h$. What we are asked to find is $\frac{dh}{dt}$. Putting the second and third equations together to eliminate r, we see that $V = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h = \frac{\pi h^3}{12}$. Differentiating both sides of this equation with respect to t, we find that

$$\begin{aligned}\frac{d}{dt}V &= \frac{d}{dt}\frac{\pi h^3}{12} \\ \frac{dV}{dt} &= \frac{dh}{dt}\frac{d}{dh}\frac{\pi h^3}{12} \\ \frac{dV}{dt} &= \frac{dh}{dt}\frac{\pi h^2}{4} \\ \frac{4\frac{dV}{dt}}{\pi h^2} &= \frac{dh}{dt}\end{aligned}$$

   and since we know $\frac{dV}{dt} = 2$ and that h is currently 10, this can be evaluated to give $\frac{8}{100\pi}$ meters per second, approximately 2.5 centimeters per second, or about half of Makoto Shinkai's sixth film.

2. **(6 points)** *Calculate $\frac{d}{ds}\arctan(4s^2)$.*

   Using the chain rule, let $u = 4s^2$, so that

$$\frac{d}{ds}\arctan u = \frac{du}{ds}\frac{d}{du}\arctan u = 8s\frac{1}{1+u^2} = \frac{8s}{1+(4s^2)^2} = \frac{8s}{1+16s^4}.$$

3. **(6 points)** *Given that $f(x) = \frac{\operatorname{arcsec} x}{\ln x}$, calculate $f'(x)$.*

   Using the quotient rule:

$$\frac{d}{dx}\frac{\operatorname{arcsec} x}{\ln x} = \frac{\ln x \frac{d}{dx}\operatorname{arcsec} x - \operatorname{arcsec} x \frac{d}{dx}\ln x}{(\ln x)^2} = \frac{\frac{\ln x}{x\sqrt{x^2-1}} - \frac{\operatorname{arcsec} x}{x}}{(\ln x)^2}$$

   It is quite possible to simplify this into a single fraction, but not really worth the trouble.

4. **(2 point bonus)** *From the implicit relationship $x^y = y^3$, determine $\frac{dy}{dx}$.*