1. **(6 points)** *Find the* general *antiderivative of the function* $f(t) = e^t - \frac{3}{t^2} + \sqrt[3]{t} - \frac{6}{\sqrt{1-t^2}}$.

   We may rewrite the middle two termsas power functions as such: $f(t) = e^t - 3t^{-2} + t^{1/3} - \frac{6}{\sqrt{1-t^2}}$. Antidifferentiating term-by-term, we get a result of $F(t) = e^t + 3t^{-1} + \frac{t^{4/3}}{4/3} - 6 \arcsin t + C$.

2. **(5 points)** *Find a function* $f(x)$ *such that* $f'(x) = 5x^3 + x - 4$ *and* $f(2) = 9$.

   Using antiderivatives, we know that $f(x)$ must have the form $\frac{5}{4}x^4 + \frac{x^2}{2} - 4x + C$; the specific function matching the given condition has a *specific* value of C. To figure out what it is, we can plug in 2 to get that

   $$9 = f(2) = \frac{5}{4} \cdot 16 + \frac{4}{2} - 4cdot2 + C = 14 + C$$

   so that $C = 9 - 14 = -5$, and in consequence $f(x) = \frac{5}{4}x^4 + \frac{x^2}{2} - 4x - 5$.

3. **(4 points)** *From the back seat of a car on a two-hour trip which started at noon, you have managed to see the speedometer once every quarter hour. Based on the information which you have compiled into the table below, estimate the distance you have traveled.*

Time	*12:00*	*12:15*	*12:30*	*12:45*	*13:00*	*13:15*	*13:30*	*13:45*	*14:00*
Speed (in miles per hour)	*10*	*45*	*60*	*50*	*80*	*70*	*75*	*50*	*10*

   Using the assumption (suitable for approximation) that the speed during each 15-minute interval is constant, we may approximate distance traveled during each time interval by the product of speed (in miles per hour) and interval length (always 0.25 hours). If we assume the speed during each interval is the speed at the beginning of that interval, we get the estimation:

   $$0.25 \times 10 + 0.25 \times 45 + 0.25 \times 60 + 0.25 \times 50 + 0.25 \times 80 + 0.25 \times 70 + 0.25 \times 75 + 0.25 \times 50$$

   or, if we use the end of the interval to determine speed, we have

   $$0.25 \times 45 + 0.25 \times 60 + 0.25 \times 50 + 0.25 \times 80 + 0.25 \times 70 + 0.25 \times 75 + 0.25 \times 50 + 0.25 \times 10$$

   but both of these evaluate to $0.25 \times 440 = 110$.

4. **(5 points)** *Write the Riemann sum* $\lim_{n\to\infty} \frac{7}{n} \sum_{i=1}^{n} \sqrt{2 + \frac{7}{n}}$ *as a definite integral.*

   Recall that the template for a Riemann sum is $\int_a^b f(x)dx = \lim_{n\to\infty} \frac{b-a}{n} \sum_{i=1}^{n} f(a + \frac{b-a}{n}i)$. It is clear that to match this template $b - a$ should be 7; the choices of $f(x)$ and a, however, are to some extent arbitrary. The most natural way to do it is to choose $f(x) = \sqrt{x}$ and $a = 2$, resulting in $b = 2+7 = 9$ and the integral $\int_2^9 \sqrt{x}dx$, but an equally valid plausible representation would be to choose $f(x) = \sqrt{2+x}$ and $a = 0$ to get $\int_0^7 \sqrt{2+x}dx$. There are actually many other ways to interpret this sum as well, such as $\int_{-4}^{3} \sqrt{x+6}dx$ or even $\int_0^{14} \frac{1}{2}\sqrt{2 + \frac{x}{2}}dx$, but those could only be discovered by willfully overcomplicating the problem.

5. **(2 point bonus)** *Without using antiderivatives, explain why for an odd function* $f(x)$ *(that is, a function for which* $f(-x) = -f(x)$*), it must be the case that* $\int_{-a}^{a} f(x) = 0$.