1. Prove that, for an integral domain R, the ring of polynomials $R[x]$ is an integral domain, and that the units in $R[x]$ are exactly the units in R.

2. Prove that the ring $\mathbb{Z}_2[x]/\langle x^3 + x + 1\rangle$ is a field, but that $\mathbb{Z}_3[x]/\langle x^3 + x + 1\rangle$ is not a field.

3. Given a ring homomorphism $\varphi : R \to R'$ on commutative rings, let $\varphi^{-1}(S) = \{r \in R : \varphi(r) \in S\}$. Note that φ^{-1} will map ideals to ideals.

   (a) Prove that if I' is a prime ideal of R', then $\varphi^{-1}(I')$ is a prime ideal of R.

   (b) Demonstrate that if I' is a maximal ideal of R', then $\varphi^{-1}(I')$ need not be a maximal ideal of R.

4. Prove that for ideals I and J of a commutative ring, and IJ representing the set consisting of every product of terms from I and J, it is the case that $IJ \subseteq I \cap J$. Under what circumstances will $IJ = I \cap J$?

5. For any subset A of a commutative ring R, the annihilator $\text{Ann}(A)$ of A is the set of all $r \in R$ such that $ra = 0$ for all $a \in A$. Prove that $\text{Ann}(A)$ is always an ideal in R.

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