1. For a field F, let $F(x)$ denote the field of fractions of the ring $F[x]$; in other words, the field of rational functions with coefficients in F. Prove that there is no element $f \in F(x)$ such that $f^2 = x$.
2. Prove that, if I is a prime ideal of a ring R, then $I[x]$ is a prime ideal of $R[x]$.
3. Describe a field of order 25 and a field of order 27.
4. Let $\alpha \in \mathbb{R}$ be a *transcendental number*, i.e., there is no nonzero polynomial with integer coefficients which has α as a zero. Prove that α^2 cannot be written as a linear combination of α and a rational number, i.e. $\alpha^2 = \alpha x + y$ for $x, y \in \mathbb{Q}$.
5. Prove that for a principal ideal domain D with p an irreducible element of D, $D/\langle p \rangle$ is a field. Show that if D is merely an integral domain, $D/\langle p \rangle$ may not be a field.

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