1. Demonstrate an example of a subring D of ring D' such that: D' is a unique factorization domain but not a field, and D is an integral domain but not a unique factorization domain.

2. Let an integral domain D be called *reverse-Noetherian* if every sequence of ideals $D \supseteq I_1 \supsetneq I_2 \supsetneq I_3 \supsetneq \cdots$ is finite. Prove that D is reverse-Noetherian if and only if D is a field.

3. Prove that if D is a principal ideal domain, then for every pair of nonzero $a, b \in D$, there is a value d such that:

   - d divides both a and b.
   - If $e \in D$ is such that e divides both a and b, then e divides d.
   - $d = ar + bs$ for some $r, s \in D$.

   Prove furthermore that d is unique up to multiplication by a unit.

   (Note that these three properties are those which, in $\mathbb{N}$, describe the greatest common divisor).

4. For a linear transformation T from a vector space V to a vector space W, the *image* of T is the set of all $w \in W$ such that $w = T(v)$ for some T, and the *kernel* of T is the set of all $v \in V$ such that $T(v) = 0$. Prove that the kernel and image of T are *subspaces* of V and W respectively.

5. Let W be a vector space of finite dimension. Prove that there are not subspaces U and V of W such that $U \cap V = \{0\}$ and $\dim U + \dim V > \dim W$.

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