1. If F has characteristic p and $a \in F$,consider the polynomial $f(x) = x^p - a \in F[x]$. Prove that $f(x)$ is either irreducible over F or $f(x)$ splits in F.

2. Let $F = \mathbb{Q}(\sqrt{2}, \sqrt{3})$. Find a polynomial $f(x) \in \mathbb{Q}[x]$ such that $F \cong \mathbb{Q}[x]/\langle f(x)\rangle$, and find a basis for F considered as a vector field over $\mathbb{Q}$.

3. Let K be an extension of field F. If $\gamma \in K$ and $a, b \in F$ with a nonzero, prove that $F(a\gamma+b) = F(\gamma)$ (note: this is set-equality, not simply isomorphism).

4. Let $\alpha \in \mathbb{R}$. Prove that $\mathbb{Q}(\alpha) \cong \mathbb{Q}(x)$ if and only if α is transcendental, i.e., if α is not the root of any polynomial with rational coefficients.

5. Prove that there is no irreducible polynomial in $\mathbb{Q}[x]$ which is zero at both $x = \sqrt{5}$ and $x = \sqrt{7}$.

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