1. Fill in the following table. You need not justify your results. In each cell, place either an X or a checkmark (✓). Place an X if the ring named in the column is not an algebra of the type named in the row, and a checkmark if the ring named in the column is an algebra of the type named in the row. Place one of these two marks in *every* cell of the table; empty cells will be automatically incorrect.

	$\mathbb{Z}[i]$	$\mathbb{Z}_6$	$\mathbb{Q}[x]$	$\mathbb{Z}_5$	$\mathbb{Z}[\sqrt{-3}]$
Principal ideal domain (PID)					
Euclidean domain					
Field					
Unique factorization domain (UFD)					

2. Let $\varphi : R \to R'$ be a surjective ring homomorphism. Prove that if R is a principal ideal domain, then R' is also a principal ideal domain.

3. (a) Prove that $\mathbb{Q}[x]/\langle x^4 - 4x^3 + 6x - 2\rangle$ is a field.
   (b) Prove that $\mathbb{Z}_5[x]/\langle x^3 + 2x^2 + 2x + 2\rangle$ is a field. What is its order?

4. Prove that for a field F, every nontrivial prime ideal of $F[x]$ is maximal.

5. An ideal $I \subseteq R$ is *finitely generated* when there is a finite set $S \subseteq I$ such that I is the smallest ideal containing S (i.e., $I = \langle S\rangle$). Prove that every ideal in R is finitely generated if and only if R is Noetherian.