1. **(20 points)** *I borrow \$1000 from the Kneecappers' Trust and Loan at an annual interest rate of 21.2%, with terms to repay the loan in full 4 years later. For each of the descriptions of the interest procedures below, calculate the quantity (in dollars) of* interest *to be paid at the end of the loan term.*

   In all three cases that follow, the problem describes an instrument whose present value P is 1000, annual interest rate r is 0.212, and the number of years under consideration t is 4; in all cases we seek the interest quantity $F - P$.

   (a) *The loan earns* simple interest*.*
   Simple interest is governed by the relationship $F = P + Prt$, so

   $$F = 1000 + 1000 \times 0.212 \times 4 = 1848$$

   and thus our final repayment is \$1848, of which $\boxed{\$848}$ is interest.

   (b) *The loan earns* annually compounding interest*.*
   Annually compounding interest is governed by the relationship $F = P(1 + r)^t$, so

   $$F = 1000(1.212)^4 \approx 2157.80$$

   and thus our final repayment is \$2157.80, of which $\boxed{\$1157.80}$ is interest.

   (c) *The loan earns* quarterly compounding interest*.*
   Periodically compounding interest is governed by the relationship $F = P\left(1 + \frac{r}{n}\right)^{nt}$, where in this case $n = 4$ for quarterly compounding, so

   $$F = 1000(1 + \tfrac{0.212}{4})^{4\times 4} \approx 2284.83$$

   and thus our final repayment is \$2284.83, of which $\boxed{\$1184.83}$ is interest.

2. **(20 points)** *How many years will it take to grow an investment of \$1200 to a value of \$1600 at an annual interest rate of 2.6% if the investment pays out:*

   Here the present value P is 1200, the future value F is 1600, the annual interest rate r is 0.026, and we wish to find the number of years t under each of the circumstances below.

   (a) *Simple interest?*
   Here $t = \frac{F-P}{Pr} = \frac{1600-1200}{1200\times 0.026} \approx 12.82$, which could be left unrounded or rounded up to $\boxed{\text{13 years.}}$

   (b) *Annually compounding interest?*
   Here $t = \frac{\log \frac{F}{P}}{\log(1+r)} = \frac{\log \frac{1600}{1200}}{\log(1.026)} \approx 11.21$, which could be left unrounded or rounded up to $\boxed{\text{12 years.}}$

3. **(12 points)** *Series Q US Treasury Bonds earn 0.76% annual interest, compounding semiannually. Each bond certificate has a face value of \$500, to which it matures over the course of 30 years. What is the fair market price for a newly issued bond certificate?*

   In this case what we seek is the present value (purchase price) of an investment of known future (matured) value; we know the future value F is 500, and we know the interest rate r is 0.0076, and the number of elapsed years t is 30; finally, the number of compounding periods per year is $n = 2$, since this compounds semiannully. Thus

$$P = \frac{F}{\left(1+\frac{r}{n}\right)^{nt}} = \frac{500}{\left(1+\frac{0.0076}{2}\right)^{2\times 30}} \approx 398.23$$

so the bond, if fairly priced will be currently sold for $\boxed{\$398.23.}$.

4. **(20 points)** *The C19 index fund is an investment which reliably earns an annual interest rate of 7.2%, compounding monthly. You have $600 invested in this fund.*

   (a) *How many months need to pass before your investment has doubled in value?*

   Here we know that this investment has an initial value P of 600, an desired future value F of 1200, and an interest rate r of 0.072 provided over twelve compounding periods per year. Our goal in this case is to determine the time t associated with these other parameters. We shall plug all the information we have into the multi-annual compounding formula for t:

   $$t = \frac{\log \frac{F}{P}}{n \log\left(1+\frac{r}{n}\right)} = \frac{\log 2}{12 \log\left(1+\frac{0.072}{12}\right)} \approx 9.656.$$

   Note the above measurement is in years, so t describes twelve times as many months, resulting in an answer of $12 \times 9.656 \approx 115.87$ which we may round up (if desired) to $\boxed{116 \text{ months.}}$

   (b) *What annually-compounding interest rate would have the same yield as this fund's monthly compounding interest rate?*

   The annual yield on an investment of this sort is

   $$\left(1+\frac{0.072}{12}\right)^{12} - 1 \approx 0.07442 = \boxed{7.442\%.}$$

   Note, for instance, that the future value of $100 put into the C19 fund for one year would be $107.44, representing a growth rate of $7.44% for the year.

5. **(12 points)** *Lagomorph Loans offers a ten-year loan of $1500 with $2500 to be repaid at the end of the loan period. What annually compounding interest rate are they charging?*

   We have a present value of $P = 1500$ growing to a future value of $F = 2500$ over a period of $t = 10$ years, and with to determine the annually compounding interest rate. We get a result of

   $$r = \sqrt[t]{\frac{F}{P}} - 1 = \sqrt[10]{\frac{2500}{1500}} - 1 \approx 0.0524 = 5.24\%$$

   so our annual interest rate is about $\boxed{5.24\%.}$

6. **(16 points+3 point bonus)** *Anna deposits $3000 in a "rewards checking" account which pays an annual rate of 4.5% compounded monthly during months when she uses their bank-branded credit card, but which only pays an annual rate of 0.5% compounded monthly in months when she does not. For the first ten months she uses their credit card but neglects to do so in for the next fourteen months.*

(a) **(16 points)** *What is the balance in her account at the end of this two-year period?*

Since the interest rate will change, we may find it easiest to to calculate an “intermediary” future value. So the first ten months of our investment is described by a present value of \$3000 with an annual interest rate of 4.5% compounding each month. At the end of that ten-month period, we could calculate the account balance to be

$$F = P\left(1+\frac{r}{n}\right)^{nt} = 3000\left(1+\frac{0.045}{12}\right)^{12\times\frac{10}{12}} = 3114.41755$$

Since this is an intermediary stage of our calculation, we want to keep it to as high a precision as possible. Then we can take this intermediary balance, and use it as an input to the new interest-bearing process which describes the remaining fourteen months. So, in order to figure out the final account balance, we would take 3114.41755 as the present value of an investment which continues for fourteen months, accruing 0.5% interest compounding monthly. We can use the same formula as above, with different parameters, to determine the eventual balance:

$$F = P\left(1+\frac{r}{n}\right)^{nt} = 3114.4175\left(1+\frac{0.005}{12}\right)^{12\times\frac{14}{12}} = 3132.63$$

so her account would have a total balance of $\boxed{\$3132.63.}$

(b) **(3 point bonus)** *What is the effective APY she has earned over these two years?*

The annually compounding interest rate (i.e., the APY) of this investment is one such that a present value of \$3000 is increased to a future value of \$3132.63 over two years, so:

$$3132.63 = 3000(1+r)^2$$

which is algebraically convertable to $r\sqrt{\frac{3132.63}{3000}} - 1 \approx 0.0219 = 2.19\%$.