1. **(16 points)** *How much would you need to invest today at 3% annual interest, compounded monthly, in order to accumulate* \$10,000 *over the next six years?*

   In this case what we seek is the present value (initial contribution) of an investment; we know the future value F is 10000, and we know the interest rate r is 0.03, and the number of elapsed years t is 6; finally, the number of compounding periods per year is 12, since this compounds monthly. Thus we need to solve for P in the equation:

$$\begin{aligned} F &= P\left(1+\frac{r}{n}\right)^{nt} \\ 10000 &= P\left(1+\frac{0.03}{12}\right)^{12\times 6} \\ \frac{10000}{\left(1+\frac{0.03}{12}\right)^{12\times 6}} &= P \end{aligned}$$

   so the present value is $\boxed{\$8354.58}$ or $\boxed{\$8354.57}$ depending on rounding.

2. **(20 points)** *You have invested* \$2,000 *in an account which pays an annual interest rate of 3.4% compounding quarterly.*

   (a) *What is the minimum number of quarters that this principal must be invested to be worth at least* \$3,200*?*

   Here we know that this investment has an initial value P of 2000, an intended future value F of 3200, and an interest rate r of 0.034 provided over four compounding periods per year. Our goal in this case is to determine the time t associated with these other parameters. We shall plug all the information we have into the multi-annaul compounding formula and solve algebraically for t:

$$\begin{aligned} F &= P\left(1+\frac{r}{n}\right)^{nt} \\ 3200 &= 2000\left(1+\frac{0.034}{4}\right)^{4t} \\ \frac{3200}{2000} &= \left(1+\frac{0.034}{4}\right)^{4t} \\ \log\frac{3200}{2000} &= \log\left(1+\frac{0.034}{4}\right)^{4t} \\ \log\frac{3200}{2000} &= 4t\log\left(1+\frac{0.034}{4}\right) \\ \frac{\log\frac{3200}{2000}}{4\log\left(1+\frac{0.034}{4}\right)} &= t \end{aligned}$$

   so using a calculator, we may compute that $t = \frac{\log\frac{3200}{2000}}{4\log\left(1+\frac{0.034}{4}\right)} \approx 13.88$. Note that t is in *years*, so in order to find a number of quarters, we'd need to multiply by 4 to get 55.53 months; since a full number of quarter is needed, we round up to $\boxed{56,}$ which will be able to be adequately sanity-checked by the second part of the problem.

   (b) *At the end of this full number of quarters, what is the actual value of your investment?*

Now, using the result from the previous part, we know we are looking at an investment of \$2,000 which is being subjected to 3.4% annual interest compounded quarterly for 56 quarters, or $\frac{56}{4}$ years. We *expect*, based on the context, that the value of the investment after this length of time will be slightly more than \$3,200, since that's exactly what our length of time was designed to do. We plug our known present value, interest rate, compounding period size, and length of time into the investment growth formula:

$$F = P\left(1 + \frac{r}{n}\right)^{nt} = 2000\left(1 + \frac{0.034}{4}\right)^{4\times\frac{56}{4}} \approx 3212.776$$

so our investment's actual value is $\boxed{\$3212.77}$ (or $\boxed{\$3212.78,}$ if the bank rounds up).

Note that the second part of this question served as a "sanity check" for the first part. The second part illustrates, by directly calculating the future value, that 56 quarters is just barely enough time to push the account's value over \$3200.

3. **(20 points)** *A man borrows \$2000 and pays off the loan three years later by paying the lender back \$2400. Find the annual interest rate associated with the loan, expressed as a percentage and rounded to the nearest hundredth of a percent, in each of the following cases:*

In both cases that follow, the problem describes an instrument whose present value P is 2000, future value F is 2400, and the number of years under consideration t is 3; in both cases we seek the interest rate r.

(a) *The loan earns* simple interest.

Simple interest is governed by the relationship $F = P + Prt$, so

$$\begin{aligned} 2400 &= 2000 + 2000 \times r \times 3 \\ 400 &= 2000 \times r \times 3 \\ \frac{400}{2000 \times 3} &= r \end{aligned}$$

so $r = \frac{400}{6000} \approx 0.06667 \approx \boxed{6.67\%.}$

(b) *The loan earns* annually compounding interest.

Annually compounding interest is governed by the relationship $F = P(1 + r)^t$, so

$$\begin{aligned} 2400 &= 2000(1 + r)^3 \\ \frac{2400}{2000} &= (1 + r)^3 \\ \left(\frac{2400}{2000}\right)^{1/3} &= [(1 + r)^3]^{1/3} \\ \left(\frac{2400}{2000}\right)^{1/3} &= (1 + r) \\ \left(\frac{2400}{2000}\right)^{1/3} - 1 &= r \end{aligned}$$

so $r = \left(\frac{2400}{2000}\right)^{1/3} - 1 \approx 0.062658 \approx \boxed{6.27\%.}$

4. **(16 points)** *A "penalty rate" loan charges 5.1% interest compounded monthly for the first year and 12.3% compounded semiannually thereafter. What would you owe if you borrowed $1100 on this loan and neglected to pay it for five and a half years?*

Since the terms of the loan change midway through, we need to calculate an "intermediary" future value. So, for instance, the first two year of our investment is described by a present value of $1100, and has an interest rate of 5.1% compounding twelve times per year. At the end of that year, we could calculate the investment to have a value of

$$F = P\left(1 + \frac{r}{n}\right)^{nt} = 1100\left(1 + \frac{0.051}{12}\right)^{12\times 1} = 1157.43009$$

Since this is an intermediary stage of our calculation, we want to keep it to as high a precision as possible. Then we can take the output of the original loan, and use it as an input to the new loan which describes the remaining four and a half years. So, in order to figure out the final value of the investment, we would take 1157.43009 as the present value of an investment which continues for 4.5 years, accruing 12.3% interest compounding semiannually. We can use the same formula as above, with different parameters, to determine the eventual value of this investment:

$$F = P\left(1 + \frac{r}{n}\right)^{nt} = 1157.43009\left(1 + \frac{0.123}{2}\right)^{4.5\times 2} = 1980.4995$$

so our loan would have increased in value to $\boxed{\$1980.50.}$

5. **(12 points)** *Sleet Bank's savings account returns 6.2% annual interest, compounded semiannually, while an account with the Haberdasher's Credit Union returns 6.1% compounded monthly. Which of these two accounts has a higher return on investment?*

This could be done by calculating the future value of two identical investments in each vehicle, but is much more easily done by comparing the APYs associated with the two different accounts. Recall that the APY for an investment with nominal annual rate r and n compounding periods per year is

$$\left(1 + \frac{r}{n}\right)^{n} - 1$$

so the Sleet Bank acoount, with $r = 0.062$ and $n = 2$, corresponds to APY

$$\left(1 + \frac{0.062}{2}\right)^{2} - 1 = 0.062961 = 6.2961\%,$$

while the HCU account has $r = 0.061$ and $n = 12$, with an APY of

$$\left(1 + \frac{0.061}{12}\right)^{12} - 1 \approx 0.062734 = 6.2734\%.$$

Since it has a higher APY, we may conclude that the $\boxed{\text{Sleet Bank account}}$ is a better investment.

6. **(16 points)** *If you invest* $3,500 *in your bank at a* 4.5% *annual interest rate, what will be the value of the account in 3 years if the bank provides:*

Here the present value P is 3500, the annual interest rate r is 0.045, and the number of years t is 3. We wish to find the future value F under several scenarios below.

(a) *Simple interest?*

Here $F = P + Prt = 3500 + 3500 \times 0.045 \times 3 = \boxed{\$3972.50}$.

(b) *Compound interest, compounded quarterly?*

Here we have compounding with 4 periods per year, so using $n = 4$ we calculate $F = P\left(1 + \frac{r}{n}\right)^{nt} = 3500\left(1 + \frac{0.045}{4}\right)^{3\times 4} \approx \boxed{\$4002.86.}$