1. **(35 points)** *Borbála buys a house which costs \$175,000. She pays* 30% *of the price down and finances the rest with a 30-year mortgage at* 5.16% *annual interest with* one point.

   (a) *What is her monthly payment? Make certain to calculate the correct loan principal first.*
   Since 30% of the house is being paid for by the down payment, only 70% is being financed; thus she needs a loan to finance $\$175000 \times 0.70 = \$122500$. However, she also needs to finance the purchase of points equal to 1% of the loan principal, so the loan principal is subject to the equation
   $$P = \$122500 + 0.01P$$
   which has solution $P = \frac{\$122500}{0.99} = \$123737.37$.
   Now, using the formula given (or an algebraic derivation), the monthly payment is
   $$A = \frac{P \times \frac{r}{n}}{\left(1 - \left(1 + \frac{r}{n}\right)^{-nt}\right)} = \frac{\$123737.37 \times 0.0043}{(1 - 1.0043^{-360})} = \boxed{\$676.40}$$

   (b) *What is the balance on the loan twelve years later, when she still has 216 payments left?*
   Here we know that the loan is a monthly loan with an annual interest rate of 5.16%, a periodic payment of \$676.40, and 216 payments left to be made. Using the formula for present value of a loan:
   $$P = A\frac{\left(1 - \left(1 + \frac{r}{n}\right)^{-nt}\right)}{\left(\frac{r}{n}\right)} = \$676.40 \times \frac{1 - 1.0043^{-216}}{0.0043} = \boxed{\$95039.88}$$
   (this answer may be \$95039.73, if intermediary rounding is used)

   (c) *Twelve years into her mortgage, when she has the balance determined in part (b), she has an opportunity to refinance, paying \$2200 in closing costs, which will be rolled into a new 30-year loan at 4.8% annual interest. Determine her new monthly payment.*
   The new mortgage will have a balance of \$97239.88 (or close thereto), an annual interest rate of 4.8%, and a 30-year lifetime. We use the same formula as in part (a) with different numbers:
   $$p = \frac{P \times \frac{r}{n}}{\left(1 - \left(1 + \frac{r}{n}\right)^{-nt}\right)} = \frac{\$97239.88 \times 0.004}{(1 - 1.004^{-360})} = \boxed{\$510.18}$$

   (d) *Based on the total paid over the remaining lifetime of the loan, does refinancing reduce her total repayment? Show your work.*
   Over the 30 years of its lifetime, the refinanced loan will require total payment of $360 \times \$510.18 = \$183666.22$ (your answer may differ slightly due to rounding), while the remaining 18 years of the original mortgage would require total payment of $216 \times \$676.40 = \$146102.63$ so $\boxed{\text{the refinanced loan is actually more expensive.}}$

2. **(12 points)** *You have bought a home theater for \$2500, and have agreed to finance it at a yearly interest rate of 20.4% compounding monthly; you will be paying off your loan with three montlhy payments. Your* ***first two*** *monthly payments will be for \$861.83 each. Using this information, complete the amortization table below for your loan.*

Month	Balance at beginning of month	Payment	Interest paid	Principal repayment	Balance at end of month
1	\$2500.00	\$861.83	\$42.50	\$819.33	\$1680.67
2	\$1680.67	\$861.83	\$28.57	\$833.26	\$847.41
3	\$847.41	\$861.82	\$14.41	\$847.41	\$0.00

3. **(24 points)** *You have taken out a small-business loan of \$20,000 at an interest rate of 6.4% compounded* quarterly. *You are paying back \$1500 each quarter.*

   (a) *How many quarters (rounded up) will it take for you to pay off the loan in its entirety?*

   Here we know all the details of a loan *except* the payoff time. In this case the balance is \$20000, the annual interest rate is 6.4%, the number of pay-periods per year is 4, and the periodic payment is 1500. We wish to find the number of quarters $4t$ in over which this loan is paid off, so we use algebraic manipulation of the formula relating all the parameters of a loan:

$$P = A\frac{\left(1 - \left(1 + \frac{r}{n}\right)^{-nt}\right)}{\left(\frac{r}{n}\right)}$$

$$\$20000 = \$1500\frac{(1 - 1.016^{-4t})}{0.016}$$

$$0.016 \times \frac{20000}{1500} = 1 - 1.016^{-4t}$$

$$1.016^{-4t} = 1 - 0.016 \times \frac{20000}{1500}$$

$$\log(1.016^{-4t}) = \log\left(1 - 0.016 \times \frac{20000}{1500}\right)$$

$$-4t\log(1.016) = \log\left(1 - 0.016 \times \frac{20000}{1500}\right)$$

$$4t = -\frac{\log\left(1 - 0.016 \times \frac{20000}{1500}\right)}{\log(1.016)} = 15.116$$

   which rounds up to $\boxed{\text{16 quarters (or 4 years).}}$

   (b) *How much interest total will you have paid on the loan over its lifetime?*

   The total payment over the life of the loan is $15.116 \times \$1500 = \$22674.86$, of which \$20000 was the principal, so the actual interest paid is $\boxed{\$2674.86.}$

4. **(16 points)** *Alice is saving money for a car; she currently has \$1250 in her "car fund" and is putting \$50 in per month for four years. Her car fund is an account earning 2.4% interest, compounded monthly. At the end of four years, how much money will there be in this account?*

   The initial investment of \$1250 will grow in four years according to the standard interest-growth formula to a future value of $\$1250\left(1 + \frac{0.024}{12}\right)^{12\times 4} = \$1375.82$. In addition, her monthly contribution of \$50 will accumulate over time, according to the accumulation-of-contribution formula, to a future value of

$$F = A\frac{\left(1 + \frac{r}{n}\right)^{nt} - 1}{\frac{r}{n}} = \$50 \times \frac{1.002^{48} - 1}{0.002} = \$2516.34$$

   so the total value of both her original investment and her periodic contribution comes out to $\$1375.82 + \$2516.34 = \boxed{\$3892.16.}$

5. **(13 points)** *Ibrahim wishes to borrow money from a bank that is offering a 6% interest rate compounded monthly. He is willing to repay \$300 per month for five years.*

(a) *How much money would the bank be willing to lend him on these terms?*
Using the formula for present value of a loan:

$$P = p\frac{\left(1-\left(1+\frac{r}{n}\right)^{-nt}\right)}{\left(\frac{r}{n}\right)} = \$300 \times \frac{1-1.005^{-60}}{0.005} = \boxed{\$15517.67}$$

(b) *How much* interest *would he pay back over the life of the loan?*
The total he pays in his sixty monthly payments will be $60 \times \$300 = \$18000$, of which \$15517.67 is principal, so the total interest is the remaining $\boxed{\$2482.33.}$

6. **(5 point bonus)** *As seen in class, for a loan with particular loan parameters P, r, and n, it is possible for the periodic payment to be so low that the loan will never be paid off. Find a formula on the back of the page for the threshold on the value of the periodic payment A at which it becomes impossible to pay off the loan, in terms of the parameters P, r, and n.*

If the principal is being paid down, no matter how little payment is being made, then *eventually* the loan will eventually be paid off. However, in a case where the prinicpal repayment is zero — or worse yet, negative — the principal will never actually be paid off. If the loan has an initial balance of P, the interest will be $P \times \frac{r}{n}$, and the principal repayment will be $A - P \times \frac{r}{n}$. If this is a positive number (even a very small one) then the loan will eventually be paid off. We see that a problem arises only if $A - P \times \frac{r}{n} \leq 0$, or if $\boxed{A \leq P\frac{r}{n}.}$

Je n'ai rien vaillant; je dois beaucoup; je donne le reste aux pauvres. [I have nothing, I owe much, and the rest I leave to the poor.] —the will of François Rabelais