1. (35 points) Borbála buys a house which costs $175,000. She pays 30\% of the price down and finances the rest with a 30-year mortgage at 5.16\% annual interest with one point.

(a) What is her monthly payment? Make certain to calculate the correct loan principal first.

Since 30\% of the house is being paid for by the down payment, only 70\% is being financed; thus she needs a loan to finance $175000 \times 0.70 = $122500. However, she also needs to finance the purchase of points equal to 1\% of the loan principal, so the loan principal is subject to the equation

\[ P = $122500 + 0.01P \]

which has solution \( P = \frac{122500}{0.99} = $123737.37 \).

Now, using the formula given (or an algebraic derivation), the monthly payment is

\[ A = \frac{P \times \frac{r}{n}}{(1 - (1 + \frac{r}{n})^{-nt})} = \frac{123737.37 \times 0.0043}{(1 - 1.0043^{-360})} = $676.40 \]

(b) What is the balance on the loan twelve years later, when she still has 216 payments left?

Here we know that the loan is a monthly loan with an annual interest rate of 5.16\%, a periodic payment of $676.40, and 216 payments left to be made. Using the formula for present value of a loan:

\[ P = A \left(1 - (1 + \frac{r}{n})^{-nt}\right) = 676.40 \times \frac{1 - 1.0043^{-216}}{0.0043} = $95039.88 \]

(this answer may be $95039.73, if intermediary rounding is used)

(c) Twelve years into her mortgage, when she has the balance determined in part (b), she has an opportunity to refinance, paying $2200 in closing costs, which will be rolled into a new 30-year loan at 4.8\% annual interest. Determine her new monthly payment.

The new mortgage will have a balance of $97239.88 (or close thereto), an annual interest rate of 4.8\%, and a 30-year lifetime. We use the same formula as in part (a) with different numbers:

\[ P = \frac{P \times \frac{r}{n}}{(1 - (1 + \frac{r}{n})^{-nt})} = \frac{97239.88 \times 0.004}{(1 - 1.004^{-360})} = $510.18 \]

(d) Based on the total paid over the remaining lifetime of the loan, does refinancing reduce her total repayment? Show your work.

Over the 30 years of its lifetime, the refinanced loan will require total payment of 360 \times $510.18 = $183666.22 (your answer may differ slightly due to rounding), while the remaining 18 years of the original mortgage would require total payment of 216 \times $676.40 = $146102.63 so the refinanced loan is actually more expensive.

2. (12 points) You have bought a home theater for $2500, and have agreed to finance it at a yearly interest rate of 20.4\% compounding monthly; you will be paying off your loan with three monthly payments. Your first two monthly payments will be for $861.83 each. Using this information, complete the amortization table below for your loan.

<table>
<thead>
<tr>
<th>Month</th>
<th>Balance at beginning of month</th>
<th>Payment</th>
<th>Interest paid</th>
<th>Principal repayment</th>
<th>Balance at end of month</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2500.00</td>
<td>$861.83</td>
<td>$42.50</td>
<td>$819.33</td>
<td>$1680.67</td>
</tr>
<tr>
<td>2</td>
<td>$1680.67</td>
<td>$861.83</td>
<td>$28.57</td>
<td>$833.26</td>
<td>$847.41</td>
</tr>
<tr>
<td>3</td>
<td>$847.41</td>
<td>$861.82</td>
<td>$14.41</td>
<td>$847.41</td>
<td>$0.00</td>
</tr>
</tbody>
</table>
3. **(24 points)** You have taken out a small-business loan of $20,000 at an interest rate of 6.4% compounded quarterly. You are paying back $1500 each quarter.

(a) **How many quarters (rounded up) will it take for you to pay off the loan in its entirety?**

Here we know all the details of a loan except the payoff time. In this case the balance is $20000, the annual interest rate is 6.4%, the number of pay-periods per year is 4, and the periodic payment is 1500. We wish to find the number of quarters \(4t\) in over which this loan is paid off, so we use algebraic manipulation of the formula relating all the parameters of a loan:

\[
P = A \left( 1 - \left( \frac{r}{n} \right)^{-nt} \right)
\]

\[
\$20000 = \$1500 \left( 1 - \frac{0.016}{4}^{-4t} \right)
\]

\[
0.016 \times \frac{20000}{1500} = 1 - 1.016^{-4t}
\]

\[
1.016^{-4t} = 1 - 0.016 \times \frac{20000}{1500}
\]

\[
\log(1.016^{-4t}) = \log \left( 1 - 0.016 \times \frac{20000}{1500} \right)
\]

\[
-4t \log(1.016) = \log \left( 1 - 0.016 \times \frac{20000}{1500} \right)
\]

\[
4t = -\frac{\log \left( 1 - 0.016 \times \frac{20000}{1500} \right)}{\log(1.016)} = 15.116
\]

which rounds up to 16 quarters (or 4 years).

(b) **How much interest total will you have paid on the loan over its lifetime?**

The total payment over the life of the loan is \(15.116 \times \$1500 = \$22674.86\), of which \$20000 was the principal, so the actual interest paid is \$2674.86.

4. **(16 points)** Alice is saving money for a car; she currently has $1250 in her “car fund” and is putting $50 in per month for four years. Her car fund is an account earning 2.4% interest, compounded monthly. At the end of four years, how much money will there be in this account?

The initial investment of $1250 will grow in four years according to the standard interest-growth formula to a future value of \(F = A \left(1 + \frac{r}{n} \right)^{nt} = 1250 \left(1 + \frac{0.024}{12} \right)^{4 \times 12} = \$1375.82\). In addition, her monthly contribution of $50 will accumulate over time, according to the accumulation-of-contribution formula, to a future value of

\[
F = A \left(1 + \frac{r}{n} \right)^{nt} = 50 \times \frac{1.002^{48} - 1}{0.002} = \$2516.34
\]

so the total value of both her original investment and her periodic contribution comes out to \$1375.82 + \$2516.34 = \$3892.16.

5. **(13 points)** Ibrahim wishes to borrow money from a bank that is offering a 6% interest rate compounded monthly. He is willing to repay $300 per month for five years.
(a) How much money would the bank be willing to lend him on these terms?

Using the formula for present value of a loan:

\[ P = p \left( \frac{1 - (1 + \frac{r}{n})^{-nt}}{\frac{r}{n}} \right) = \$300 \times \frac{1 - 1.005^{-60}}{0.005} = \$15517.67 \]

(b) How much interest would he pay back over the life of the loan?

The total he pays in his sixty monthly payments will be \(60 \times \$300 = \$18000\), of which \$15517.67 is principal, so the total interest is the remaining \$2482.33.

6. (5 point bonus) As seen in class, for a loan with particular loan parameters \(P, r,\) and \(n,\) it is possible for the periodic payment to be so low that the loan will never be paid off. Find a formula on the back of the page for the threshold on the value of the periodic payment \(A\) at which it becomes impossible to pay off the loan, in terms of the parameters \(P, r,\) and \(n.\)

If the principal is being paid down, no matter how little payment is being made, then eventually the loan will eventually be paid off. However, in a case where the principal repayment is zero — or worse yet, negative — the principal will never actually be paid off. If the loan has an initial balance of \(P,\) the interest will be \(P \times \frac{r}{n},\) and the principal repayment will be \(A - P \times \frac{r}{n} \). If this is a positive number (even a very small one) then the loan will eventually be paid off. We see that a problem arises only if \(A - P \times \frac{r}{n} \leq 0,\) or if \(A \leq P \frac{r}{n} \).

Je n’ai rien vaillant; je dois beaucoup; je donne le reste aux pauvres. [I have nothing, I owe much, and the rest I leave to the poor.] —the will of François Rabelais