

1. **(7 points)** *You have \$1500 in an account that pays an annual interest rate of 4.2%, compounded monthly.*

In both parts below, the present value is our initial deposit $P = 1500$. The annual interest rate is $r = 0.042$, and the number of compounding periods per year is $n = 12$, since the compounding is monthly.

- (a) *How many months will it take your account balance to grow from this initial balance to \$2000?*

Here we desire an eventual balance $F = 2000$, after an unknown but sought length of time t . Using the formula for time to a desired future value for a periodically compounding instrument:

$$t = \frac{\log \frac{F}{P}}{n \log \left(1 + \frac{r}{n}\right)} = \frac{\log \frac{2000}{1500}}{12 \log \left(1 + \frac{0.042}{12}\right)} \approx 6.86155 \text{ years}$$

Since the answer above is in years, we would need to multiply by 12 to get the number of months; thus t is 82.3386 months, which would sensibly be rounded up to 83 months or alternatively 6 years and 11 months.

- (b) *How many months will it take your account balance to grow from the initial balance to \$3000?*

The only change from above is that $F = 3000$ in this case, so

$$t = \frac{\log \frac{F}{P}}{n \log \left(1 + \frac{r}{n}\right)} = \frac{\log \frac{3000}{1500}}{12 \log \left(1 + \frac{0.042}{12}\right)} \approx 16.5323 \text{ years}$$

which equivalently would be 198.388 months, sensibly rounded up to 199 months or alternatively 16 years and 7 months.

2. **(6 points)** *I am depositing \$800 at the end of each quarter into an initially empty retirement account which earns 2.4% annual interest compounded quarterly. What will the balance of the account be after 7 years and 3 months have passed? Of the total balance at this time, what quantity is the result of interest earned? Circle both answers, indicating which is which.*

Here we have an investment accumulating value over time, subject to the formula $F = A \frac{(1+i)^m - 1}{i}$. In this scenario, our periodic payment is $A = 800$, our quarterly interest rate is $\frac{0.024}{4} = 0.006$, and our total number of quarters is $(7 + \frac{3}{12}) \times 4 = 29$. Thus:

$$F = 800 \times \frac{1.006^{29} - 1}{0.006} \approx 25258.27$$

so our final balance is \$25,258.27. Note that we made 29 contributions of \$800, so we actually provided $29 \times \$800 = \23200 of that balance. The remainder is interest earned, so we have earned a total of $\$25258.27 - \$23200 = \text{\$2058.27}$ in interest.

3. **(7 points)** *I am taking out a 48-month car loan at an annual interest rate of 3.375% compounded monthly. I can afford a monthly payment of \$400 on this loan. How large a loan principal could I take out? What is the finance charge (i.e., total amount of interest paid over the life of the loan)? Label which answer is which.*

In this case our instrument (a loan) is having its principal *depleted* over time, so we use the formula $P = A \frac{1-(1+i)^{-m}}{i}$. In this scenario, our periodic payment is $A = 400$, our quarterly interest rate is $\frac{0.03375}{12} = 0.0028125$, and our total number of months is 48. Thus:

$$P = 400 \times \frac{1 - 1.0028125^{-48}}{0.0028125} \approx 17936.86$$

so our present value (which is to say, the initial loan principal) is $\boxed{\$17,936.86}$. In paying this loan, we will make 48 payments of \$400, so we pay back $48 \times \$400 = \$19,200$. Much of this was our original loan principal, but the overage is the finance charge, which will be $\$19200 - \$17936.86 = \boxed{\$1263.14}$