

1. **(7 points)** You have \$1500 in an account that pays an annual interest rate of 4.2%, compounded monthly.

In both parts below, the present value is our initial deposit $P = 1500$. The annual interest rate is $r = 0.042$, and the number of compounding periods per year is $n = 12$, since the compounding is monthly.

- (a) How many months will it take your account balance to grow from this initial balance to \$2000?

Here we desire an eventual balance $F = 2000$, after an unknown but sought length of time t . Using the formula for time to a desired future value for a periodically compounding instrument:

$$t = \frac{\log \frac{F}{P}}{n \log \left(1 + \frac{r}{n}\right)} = \frac{\log \frac{2000}{1500}}{12 \log \left(1 + \frac{0.042}{12}\right)} \approx 6.86155 \text{ years}$$

Since the answer above is in years, we would need to multiply by 12 to get the number of months; thus t is 82.3386 months, which would sensibly be rounded up to 83 months or alternatively 6 years and 11 months.

- (b) How many months will it take your account balance to grow from the initial balance to \$3000?

The only change from above is that $F = 3000$ in this case, so

$$t = \frac{\log \frac{F}{P}}{n \log \left(1 + \frac{r}{n}\right)} = \frac{\log \frac{3000}{1500}}{12 \log \left(1 + \frac{0.042}{12}\right)} \approx 16.5323 \text{ years}$$

which equivalently would be 198.388 months, sensibly rounded up to 199 months or alternatively 16 years and 7 months.

2. **(6 points)** I am depositing \$800 at the end of each quarter into an initially empty retirement account which earns 2.4% annual interest compounded quarterly. What will the balance of the account be after 7 years and 3 months have passed? Of the total balance at this time, what quantity is the result of interest earned? Circle both answers, indicating which is which.

Here we have an investment accumulating value over time, subject to the formula $F = A \frac{(1+i)^m - 1}{i}$. In this scenario, our periodic payment is $A = 800$, our quarterly interest rate is $\frac{0.024}{4} = 0.006$, and our total number of quarters is $(7 + \frac{3}{12}) \times 4 = 29$. Thus:

$$F = 800 \times \frac{1.006^{29} - 1}{0.006} \approx 25258.27$$

so our final balance is \$25,258.27. Note that we made 29 contributions of \$800, so we actually provided $29 \times \$800 = \23200 of that balance. The remainder is interest earned, so we have earned a total of $\$25258.27 - \$23200 = \text{\$2058.27}$ in interest.

3. **(7 points)** I am taking out a 48-month car loan at an annual interest rate of 3.375% compounded monthly. I can afford a monthly payment of \$400 on this loan. How large a loan principal could I take out? What is the finance charge (i.e., total amount of interest paid over the life of the loan)? Label which answer is which.

In this case our instrument (a loan) is having its principal *depleted* over time, so we use the formula $P = A \frac{1-(1+i)^{-m}}{i}$. In this scenario, our periodic payment is $A = 400$, our quarterly interest rate is $\frac{0.03375}{12} = 0.0028125$, and our total number of months is 48. Thus:

$$P = 400 \times \frac{1 - 1.0028125^{-48}}{0.0028125} \approx 17936.86$$

so our present value (which is to say, the initial loan principal) is $\boxed{\$17,936.86}$. In paying this loan, we will make 48 payments of \$400, so we pay back $48 \times \$400 = \$19,200$. Much of this was our original loan principal, but the overage is the finance charge, which will be $\$19200 - \$17936.86 = \boxed{\$1263.14}$