This test is closed-book and closed-notes. No calculator is allowed for this test. For full credit show all of your work (legibly!), unless otherwise specified.

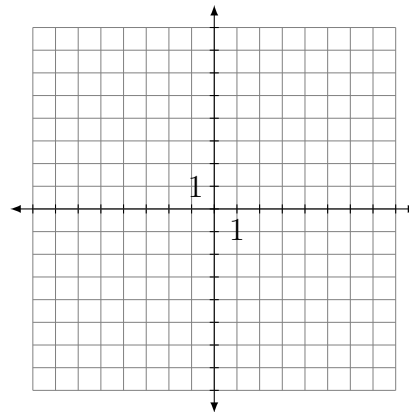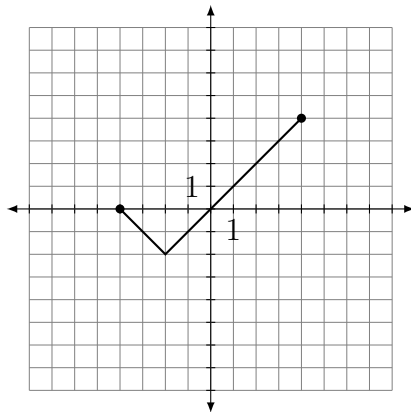1. **(10 points)** Answer the following questions about graphs.

   (a) **(3 points)** Given the piecewise function $g(x) = \begin{cases} x^2 - 2 \text{ if } & x > 1 \\ 2x - 5 \text{ if } & x \leq 1 \end{cases}$, calculate the following values:

   - $g(3)$.

   - $g(2)$.

   - $g(1)$.

   (b) **(4 points)** For $f(x)$ as shown on the graph, sketch the graph of its transformation $g(x) = 2f(-x) - 1$.

   (c) **(3 points)** Determine the equation of the line through the points $(1, 4)$ and $(4, -5)$.

1	/10
2	/ 6
3	/ 4
4	/15
5	/15
6	/ 5
7	/ 6
Σ	/60

2. **(6 points)** The Hong Kong Cavaliers have 1000 fans who would come to a concert if tickets cost \$10. Polling of the fanbase suggests that every increase in the ticket price by \$1 would reduce attendance by 50.

   (a) **(4 points)** Express the number of concertgoers as a function of the ticket price.

   (b) **(2 points)** Express the total revenue from ticket sales as a function of the ticket price.

3. **(4 points)** Alice is 30 years old and her son is currently three years old; she notes with interest that her age is currently ten times his, and would like to know how many years it will be until she is only four times as old as he is; algebraically determine how long it will take.

4. **(15 points)** Answer the following questions about the functions $f(x) = \frac{2}{x+1}$ and $g(x) = \frac{x}{x+2}$. In each question asking for multiple answers, *label which is which.*

   (a) **(3 points)** Find the inverse of the function $f(x)$.

   (b) **(2 points)** Write formulas, which need not be simplified, for $(g - f)(x)$ and $\frac{f}{g}(x)$.

   (c) **(3 points)** Write formulas, which need not be simplified, for $f(g(x))$ and $f(f(x))$.

   (d) **(3 points)** Determine the domains of $f(x)$ and $g(x)$.

   (e) **(4 points)** Determine the domains of $(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$, and $\frac{f}{g}(x)$.

5. **(15 points)** Perform the following arithmetic and algebraic operations.

   (a) **(3 points)** Factor the quadratic $x^2 - 8x + 12$.

   (b) **(3 points)** Expand and simplify the polynomial $(x^3 + 5) - (2x - 1)(x^2 + 3x)$.

   (c) **(3 points)** Calculate $16^{3/4}$.

   (d) **(3 points)** Simplify the expression $\frac{x}{x-4} - \frac{3}{x+6}$.

   (e) **(3 points)** Simplify the expression $\frac{(2x^3)^2(3x^4)}{(x^3)^4}$.

6. **(10 points)** Answer the following questions about the quadratic $q(x) = 6x^2 + 12x - 5$.

    (a) **(2 points)** What is the average rate of change of the function $q(x)$ between the points $x = -1$ and $x = 1$?

    (b) **(3 points)** Determine the x-intercepts of this quadratic if they exist (explicitly stating if they do not exist), and its y-intercept. Label which is which.

7. **(6 points)** The moon is about 240,000 miles from the earth. A rocket going to the moon goes a constant speed after launch, and then fires a secondary booster 40,000 miles into the trip to increase its speed by 8000 mph. If we want the rocket to reach the moon 30 hours after launch, how quickly does it need to be going initially?