1. **(10 points)** *Answer the following questions concerning logarithms.*

   (a) **(4 points)** *Find the domain of the function* $f(x) = \frac{\sqrt{x}}{\log_5(5-x)}$.
   There are in fact three restrictions on this function: first, the parameter of the square root must be non-negative, second, the parameter of the logarithm must be positive, and third, the denominator of the fraction must be nonzero. Thus, we require that $x \geq 0$, $5 - x > 0$, and $\log(5 - x) \neq 0$. The first and last requirements are easily simplified to $0 \leq x < 5$. The last requirement is that $5 - x \neq 1$ so $x \neq 4$. Thus, our domain is $0 \leq x < 5$ with $x \neq 4$, or, in interval form, $[0, 4) \cup (4, 5)$.

   (b) **(2 points)** *Determine the value of* $\log_2 14 - 2\log_2 6 + \log_2 \frac{9}{7}$.
   Using the various logarithm rules:

$$\log_2 14 - 2\log_2 6 + \log_2 \frac{9}{7} = \log_2 14 - \log_2(6^2) + \log_2 \frac{9}{7} = \log_2 \left(\frac{14}{6^2} \cdot \frac{9}{7}\right) = \log_2 \frac{1}{2} = -1.$$

   (c) **(4 points)** *Solve for* x *in the exponential equation* $2 \cdot 5^{(x^2+x)} = 50$.
   Dividing both sides by 2, we see that $5^{(x^2+x)} = 25$. Since 25 is the square of 5, the exponent there must be 2, so $x^2 + x$ must equal 2. Thus $x^2 + x - 2 = 0$, which can be solved with either the quadratice formula or factorization to have solutions $x = 1$ and $x = -2$.

2. **(7 points)** *Answer the following questions about the quadratic* $q(x) = 6x^2 + 12x - 5$.

   (a) **(3 points)** *Put the quadratic* $q(x)$ *in standard form.*

$$\begin{aligned} q(x) &= 6x^2 + 12x - 5 \\ &= 6(x^2 + 2x) - 5 \\ &= 6(x^2 + 2x + 1 - 1) - 5 \\ &= 6(x^2 + 2x + 1) - 6 - 5 \\ &= 6(x + 1)^2 - 11 \end{aligned}$$

   (b) **(1 point)** *Does* $q(x)$ *have a maximum or minimum value? If so, identify which it is and what its value is.*
   Its vertex is at $(-1, -11)$; since this is a quadratic with positive a, it curves upwards and this vertex is the lowest point on the graph; thus -11 is the minimum value achieved by $q(x)$.

   (c) **(3 points)** *Determine the* x*-intercepts of this quadratic if they exist (explicitly stating if they do not exist), and its* y*-intercept. Label which is which.*
   The y-intercept is simply $q(0) = 6 \cdot 0^2 + 12 \cdot 0 - 5 = -5$. The x-intercepts are given by finding the solutions to the equation $q(x) = 0$; since this equation is $6x^2 + 12x - 5 = 0$, it is easily solved using the quadratic equation:

$$x = \frac{-12 \pm \sqrt{12^2 - 4 \cdot 6(-5)}}{2 \cdot 6} = \frac{-12 \pm \sqrt{264}}{12}$$

   This can, but need not, be simplified to $1 \pm \frac{\sqrt{66}}{6}$.

3. **(10 points)** *Answer the following questions about polynomial functions.*

   (a) **(2 points)** *Identify all the* potential *rational roots of* $3x^3 + 7x - 4$*. Do not check which are actual roots.*

   By the Rational Root Theorem, the possible numerators are 1, 2, and 4, and the denominators 1 and 3. Since any combination can appear with either sign, there are 12 potential roots: 1, -1, 2, -2, 4, -4, $\frac{1}{3}$, $\frac{-1}{3}$, $\frac{2}{3}$, $\frac{-2}{3}$, $\frac{4}{3}$, and $\frac{-4}{3}$.

   Incidentally, none of these are actually roots of this particular polynomial, which has two complex roots and one irrational real root.

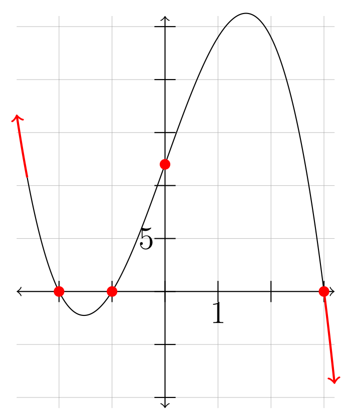
   (b) **(4 points)** *Identify the x-intercepts, y-intercept, and long-term behavior of the polynomial $f(x) = -2(x-3)(x+1)(x+2)$.*

   The y-intercept is $f(0) = (-2)(0-3)(0+1)(0+2) = 12$; if you prefer to name a point in the plane, it may be denoted $(0, 12)$.

   The x-intercepts occure when $-2(x-3)(x+1)(x+2) = 0$, which are, based on when each factor is zero, the points $x = 3$, $x = -1$, and $x = -2$. If you prefer to name points in the plane, they may be denoted $(3, 0)$, $(-1, 0)$, and $(-2, 0)$.

   The leading term of the polynomial is $-2x^3$; this is a monomial of odd degree (3) and negative coefficient (-2), so as $x \to +\infty$, $f(x) \to -\infty$, and as $x \to -\infty$, $f(x) \to +\infty$.

   For purposes of illustration, here is a plot of the curve in question, with the relevant parts discovered above appearing in red.



   (c) **(4 points)** *Using either synthetic or long division, find the quotient and remainder of the operation $\frac{4x^3-3x+5}{x+2}$.*

   Here is the result of a long division, with a quotient of $4x^2 - 8x + 13$ and a remainder of -21:

$$
\begin{array}{r|rrrr}
 & 4x^2 & -8x & +13 & \\
\hline
x+2 & 4x^3 & & -3x & +5 \\
 & -4x^3 & -8x^2 & & \\
\hline
 & & -8x^2 & -3x & \\
 & & 8x^2 & +16x & \\
\hline
 & & & 13x & +5 \\
 & & & -13x & -26 \\
\hline
 & & & & -21
\end{array}
$$

   Alternatively, we could perform synthetic division:

$$\begin{array}{r|rrrr} -2 & 4 & 0 & -3 & 5 \\ & & -8 & 16 & -26 \\ \hline & 4 & -8 & 13 & -21 \end{array}$$

and the first three entries of the lowest row give us the coefficients in the quotient $4x^2 - 8x + 13$, while the last entry is the remainder -21.

4. **(12 points)** *Solve the inequality* $\frac{1}{x+2} + \frac{3}{x-3} \leq \frac{4}{x}$.

We will move the entire expression to the left side of the inequality, and make a single fraction with a common denominator and a factored numerator:

$$\begin{aligned} \frac{1}{x+2} + \frac{3}{x-3} &\leq \frac{4}{x} \\ \frac{1}{x+2} + \frac{3}{x-3} - \frac{4}{x} &\leq 0 \\ \frac{(x-3)x + 3(x+2)x - 4(x-3)(x+2)}{(x+2)(x-3)x} &\leq 0 \\ \frac{(x^2-3x) + (3x^2+2x) - (4x^2-4x-24)}{(x+2)(x-3)x} &\leq 0 \\ \frac{3x+24)}{(x+2)(x-3)x} &\leq 0 \end{aligned}$$

We are interested, then, in when $\frac{3x+24)}{(x+2)(x-3)x}$ is nonpositive. It clearly has sign transitions when either the numerator is positive (at $x = -8$) or when the denominator is negative (at $x = -2$, $x = 0$, and $x = 3$). The expression as a whole has long-term behavior as $\frac{3x}{x^3}$, and will be positive when x is very large in either direction, so we know this expression is positive on the interval $(-\infty, -8)$, negative on $(-8, -2)$, positive on $(-2, 0)$, negative on $(0, 3)$, and positive again on $(3, \infty)$. Thus the intervals where it is negative are $(-8, -2)$ and $(0, 3)$; but we need to include the place where the expression is zero because the inequality is nonstrict, and so we get $[-8, -2) \cup (0, 3)$ as our answer.

5. **(10 points)** *Answer the following questions about growth and decay.*

   (a) **(3 points)** *The temperature in degrees Fahrenheit of a glass of cold milk t minutes after it was removed from the fridge is given by the function $f(t) = 70 - 30e^{-0.02t}$. How long will it take the milk to warm up to $55°F$?*

   We want a solution to the exponential equation $70 - 30e^{-0.02t} = 55$. Performing appropriate arithmetic on both sides of the equality, we can eventually liberate the value of t:

$$\begin{aligned}
70 - 30e^{-0.02t} &= 55 \\
-30e^{-0.02t} &= -15 \\
e^{-0.02t} &= \frac{-15}{-30} \\
\ln e^{-0.02t} &= \ln \frac{-15}{-30} \\
-0.02t &= \ln \frac{1}{2} \\
t &= \frac{\ln \frac{1}{2}}{-0.02}
\end{aligned}$$

For reference, this value is about 34 minutes, although you could not reasonably determine that without a calculator.

(b) **(3 points)** *The radioactive alloy Cobalt-Thorium-G has a half-life of 93 years. Produce a function describing the quantity of Co-Th-G remaining in a 75 gram sample after t years.*
After t years, the sample will have halved in amount present $\frac{t}{93}$ times, so the function for the amount remaining is $f(t) = 75 \cdot \left(\frac{1}{2}\right)^{t/93}$.

(c) **(4 points)** *The environmental safety rating for Cobalt-Thorium-G indicates that unshielded human exposure will again be safe after a 75 gram sample has decayed down to 5 grams. Using your function from the previous part of this question, how many years will this take?*
We want a solution to the equation $75 \cdot \left(\frac{1}{2}\right)^{t/93} = 5$. Performing appropriate arithmetic on both sides of the equality, we can eventually liberate the value of t:

$$\begin{aligned}
75 \cdot \left(\frac{1}{2}\right)^{t/93} &= 5 \\
\left(\frac{1}{2}\right)^{t/93} &= \frac{5}{75} \\
\log_{1/2} \left(\frac{1}{2}\right)^{t/93} &= \log_{1/2} \frac{1}{15} \\
\frac{t}{93} &= \log_{1/2} \frac{1}{15} \\
t = 93 \log_{1/2} \frac{1}{15} &= \frac{\ln \frac{1}{15}}{\ln \frac{1}{2}} = \frac{\ln 15}{\ln 2}
\end{aligned}$$

The very last step is a simplification, not entirely necessary for your work. For reference, however, this answer is about 363 years.

6. **(10 points)** *Answer the following questions preparatory to sketching the rational function* $h(x) = \frac{3(x-1)(x+1)}{x+2}$.

(a) **(2 points)** *What is the function's domain?*

Since the denominator is zero when $x = -2$, our domain is where $x \neq 2$. If put in interval form, this is $(-\infty, -2) \cup (-2, \infty)$.

(b) **(2 points)** *Does this function have x-intercepts, and if so, what are they?*

The numerator is zero when $x = \pm 1$; both of these are within the domain, so they are both places where $h(x) = 0$. Thus they are x-intercepts.

(c) **(2 points)** *Where are this function's vertical asymptotes?*

The denominator is zero and the numerator nonzero at $x = 2$, so $x = 2$ is a vertical asymptote of this function.

(d) **(3 points)** *How does this function behave as x becomes very large? How does it behave as x becomes very highly negative? Label which is which.*

If we expand the numerator, the function could be written as $h(x) = \frac{3x^2-3}{x+2}$. For x of very large magnitude, the dominant terms in the numerator and denominator make this approximately $\frac{3x^2}{x} = 3x$. As $x \to +\infty$, this expression also approaches $+\infty$; likewise, as $x \to -\infty$, this expression will approach $-\infty$.

(e) **(1 point)** *Does this function have a maximum or minimum value? Why or why not?*

It has no maximum or minimum for two reasons: the asymptotic behavior induces arbitrarily-large magnitude positive and negative values close to the asymptote, and the long-term behaviors induce arbitrarily large-magnitude positive and negative values for the function at large-magnitude values of x.