

This test is closed-book and closed-notes. No calculator is allowed for this test. For full credit show all of your work (legibly!), unless otherwise specified. Answers should be simplified down to arithmetic expressions whenever possible – only unsimplifiable exponentials and common or natural logarithms may be left unevaluated.

1. **(10 points)** Answer the following questions concerning logarithms.

(a) **(4 points)** Find the domain of the function  $f(x) = \frac{\sqrt{x}}{\log_5(5-x)}$ .

(b) **(2 points)** Determine the value of  $\log_2 14 - 2\log_2 6 + \log_2 \frac{9}{7}$ .

(c) **(4 points)** Solve for  $x$  in the exponential equation  $2 \cdot 5^{(x^2+x)} = 50$ .

2. **(7 points)** Answer the following questions about the quadratic  $q(x) = 6x^2 + 12x - 5$ .

(a) **(3 points)** Put the quadratic  $q(x)$  in standard form.

(b) **(1 point)** Does  $q(x)$  have a maximum or minimum value? If so, identify which it is and what its value is.

(c) **(3 points)** Determine the  $x$ -intercepts of this quadratic if they exist (explicitly stating if they do not exist), and its  $y$ -intercept. Label which is which.

3. **(10 points)** Answer the following questions about polynomial functions,
- (a) **(2 points)** Identify all the *potential* rational roots of  $3x^3 + 7x - 4$ . Do not check which are actual roots.
- (b) **(4 points)** Identify the  $x$ -intercepts,  $y$ -intercept, and long-term behavior of the polynomial  $f(x) = -2(x - 3)(x + 1)(x + 2)$ .
- (c) **(4 points)** Using either synthetic or long division, find the quotient and remainder of the operation  $\frac{4x^3 - 3x + 5}{x + 2}$ .
4. **(12 points)** Solve the inequality  $\frac{1}{x+2} + \frac{3}{x-3} \leq \frac{4}{x}$ .

5. **(10 points)** Answer the following questions about growth and decay.
- (a) **(3 points)** The temperature in degrees Fahrenheit of a glass of cold milk  $t$  minutes after it was removed from the fridge is given by the function  $f(t) = 70 - 30e^{-0.02t}$ . How long will it take the milk to warm up to  $55^\circ\text{F}$ ?
- (b) **(3 points)** The radioactive alloy Cobalt-Thorium-G has a half-life of 93 years. Produce a function describing the quantity of Co-Th-G remaining in a 75 gram sample after  $t$  years.
- (c) **(4 points)** The environmental safety rating for Cobalt-Thorium-G indicates that unshielded human exposure will again be safe after a 75 gram sample has decayed down to 5 grams. Using your function from the previous part of this question, how many years will this take?

6. **(10 points)** Answer the following questions preparatory to sketching the rational function

$$h(x) = \frac{3(x-1)(x+1)}{x+2}.$$

(a) **(2 points)** What is the function's domain?

(b) **(2 points)** Does this function have  $x$ -intercepts, and if so, what are they?

(c) **(2 points)** Where are this function's vertical asymptotes?

(d) **(3 points)** How does this function behave as  $x$  becomes very large? How does it behave as  $x$  becomes very highly negative? Label which is which.

(e) **(1 point)** Does this function have a maximum or minimum value? Why or why not?