

This test is closed-book and closed-notes. No calculator is allowed for this test. For full credit show all of your work (legibly!), unless otherwise specified.

The problems are in no particular order, and it is suggested that you look at all of them before beginning to answer any.

1. **(10 points)** Chrome Koran's latest album, made available for purchase online, would be bought by 4500 people at a price point of \$5. Fan polling suggests that every increase in the price by \$1 would reduce the number of downloads by 500.

(a) **(3 points)** Find a function describing the *demand* for the album as a function of price.

(b) **(3 points)** Find a function describing the total *revenue* from album sales as a function of price.

(c) **(4 points)** Find a sale price for the album which maximizes revenue, and the total revenue earned at this price. Label which is which.

1	/10
2	/14
3	/ 6
4	/10
5	/10
6	/15
7	/13
8	/12
Σ	/90

2. **(14 points)** Answer the following questions about the polynomial function $f(x) = 2x^3 + 5x^2 + x - 2$.
- (a) **(3 points)** What is the average rate of change of this function between the values $x = 0$ and $x = 2$?
- (b) **(3 points)** What are all the *potential* rational zeroes of this function?
- (c) **(4 points)** Factor the polynomial into linear terms.
- (d) **(4 points)** What are the x -intercepts, y -intercept, and long term behavior of the function? Label which is which.
3. **(6 points)** Calculate the following trigonometric expressions.
- (a) **(2 points)** $\arcsin \frac{\sqrt{3}}{2}$.
- (b) **(2 points)** $\csc \frac{17\pi}{6}$
- (c) **(2 points)** $\tan \frac{-5\pi}{3}$.

4. (10 points) Answer the following questions about logarithms.

(a) (3 points) Find a value of x such that $3 \cdot 2^{2x-1} + 5 = 53$.

(b) (3 points) Calculate the value of the expression $\log_5 140 + \log_5 \frac{2}{7} - 3 \log_5 10$ exactly.

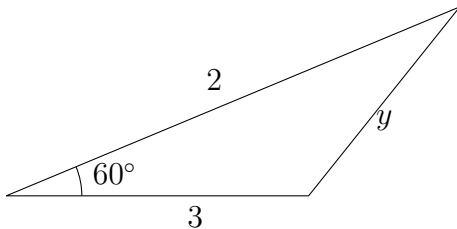
(c) (4 points) Calculate the following logarithms exactly, giving numerical answers:

- $\log_4 8$.
- $\log_7 49$
- $\log_3 \frac{1}{27}$.
- $\log_5 5$

5. (10 points) Answer the following trigonometric questions.

(a) (4 points) Simplify the expression $\tan(\operatorname{arcsec} x)$ to a form which does not use trigonometric functions.

(b) (3 points) Find the value of y in the triangle (not drawn to scale) below.



(c) (3 points) If θ describes a point in quadrant III and $\cos \theta = \frac{-2}{3}$, what is $\sin(2\theta)$?

6. **(15 points)** Answer the following questions about the functions $f(x) = \frac{\log_2(x+1)}{3x-2}$ and $g(x) = \sqrt{x-1}$. In each question asking for multiple answers, *label which is which*.
- (a) **(3 points)** Write formulas, which need not be simplified, for $f(g(x))$ and $g(g(x))$.
- (b) **(3 points)** Determine the domains of $f(x)$ and $g(x)$.
- (c) **(2 points)** Write formulas, which need not be simplified, for $(fg)(x)$ and $(f+g)(x)$.
- (d) **(4 points)** Determine the domains of $(f+g)(x)$ and $\frac{f}{g}(x)$.
- (e) **(3 points)** Find the inverse of the function $g(x)$.

7. (13 points) Answer the following questions about growth and decay.
- (a) (2 points) An investment in the Kék Rózsa fund will increase in value by 3% each year. What will the value of a \$1000 initial investment be in 2 years?
- (b) (2 points) Solarbonite has a radioactive half-life of 50 hours. Produce a function describing the quantity of solarbonite remaining in a 20-gram sample after t hours.
- (c) (3 points) Unshielded human exposure to quantities of solarbonite in excess of 8 grams is generally regarded as unsafe. How long will one need to wait until a 20-gram sample of solarbonite reaches a safe level?
- (d) (3 points) The population of Santa Carla is currently 12000. Over the course of each year the population will grow by 2%. How many years will it take for the population to reach 20000?
- (e) (3 points) A frozen pizza is taken out of a freezer and placed into a hot oven; its temperature in degrees Fahrenheit t minutes after being moved is $f(t) = 400 - 380e^{-0.03t}$. What is the original temperature of the pizza and the temperature of the oven, and how long will it take to heat to 200°F? Label each of your answers.

8. (12 points) The following twelve graphs are of the following functions:

$$\begin{array}{llll}
 A(x) = \ln x & B(x) = 2^x & C(x) = \frac{x}{(x-1)(x+2)} & D(x) = \frac{(x-1)(x+2)}{x} \\
 E(x) = 3 \cos x & F(x) = \cos \frac{x}{3} & G(x) = \cos(3x) & H(x) = \frac{1}{3} \cos x \\
 I(x) = 1 + \sin x & J(x) = \left(\frac{1}{3}\right)^x & K(x) = \cot x & L(x) = \csc 2x
 \end{array}$$

Label each picture with the letter of the appropriate function.

