1. **(5 points)** *Answer the following questions about exponents:*

   (a) **(3 points)** *Simplify the expression* $\frac{(x^2y)^3}{y^2}$.
   Using both double-exponentiation and division rules:

   $$\frac{(x^2y)^3}{y^2} = \frac{x^{3\cdot 2}y^3}{y^2} = x^6y^{3-2} = x^6y.$$

   (b) **(2 points)** *Determine the value (writing as an integer or a fraction as appropriate) of* $9^{-3/2}$.

   Note that a negative exponent denotes a reciprocal, while a fractional exponent denotes a radical of the appropriate power, so:

   $$9^{-3/2} = \frac{1}{(9^{1/2})^3} = \frac{1}{\sqrt{9}^3} = \frac{1}{3^3} = \frac{1}{27}.$$

2. **(8 points)** *Simplify the following rational expressions:*

   (a) **(4 points)** $\frac{x+3}{\frac{2x^2-17x}{4x-2}}$.

   Note that division by a fraction is indentical to multiplication by its reciprocal, so:

   $$\frac{x+3}{\frac{2x^2-17x}{4x-2}} = (x+3)\left(\frac{4x-2}{2x^2-17x}\right) = \frac{(x+3)(4x-2)}{2x^2-17x} = \frac{4x^2+12x-2x-6}{2x^2-17x} = \frac{5x^2+10x-6}{2x^2-17x}.$$

   (b) **(4 points)** $(x^2-2) - \frac{x+1}{x-1}$.

   When subtracting two rational expressions, we need to induce a common denominator, in this case $x-1$:

   $$\begin{aligned}(x^2-2) - \frac{x+1}{x-1} &= \frac{(x^2-2)(x-1)}{x-1} - \frac{x+1}{x-1} \\ &= \frac{(x^2-2)(x-1)-(x+1)}{x-1} \\ &= \frac{(x^3-x^2-2x+2)-(x+1)}{x-1} = \frac{x^3-x^2-3x+1}{x-1}\end{aligned}$$

3. **(7 points)** *A boat is traveling on a river with a current of 3 miles per hour; thus, when going upstream, the boat travels 3 mph slower than its speed in still water and when going downstream it travels 3 mph faster. This boat goes 4 miles up the river and then 4 miles back down to its starting point in a total of one hour. How fast is the boat in still water?*

   Our quantity of interest is the boat's still-water speed; let's call that x. Then its upstream journey is a distance of 4 miles at a speed of $x-3$ mph, and its downstream journey is a distance of 4 miles at a speed of $x+3$ mph, while the journey as a whole takes one full hour. We can lay this information out in a table:

	Time	Speed	Distance
Upstream		$x-3$ mph	4 miles
Downstream		$x+3$ mph	4 miles
Overall	1 hour		

Note that distances in each row are products of times and speeds, and that the overall distance and time is the sum of the distances and times respectively for the two legs of the journey (note the same is *not* true of speeds!). On this basis we can populate the rest of the table to get the following:

	Time	Speed	Distance
Upstream	$\frac{4}{x-3}$ hours	$x-3$ mph	4 miles
Downstream	$\frac{4}{x+3}$ hours	$x+3$ mph	4 miles
Overall	1 hour	8 mph	8 miles

And, since the total of the time spent in the upstream and downstream legs must be a full hour, we get the equation:

$$\frac{4}{x-3} + \frac{4}{x+3} = 1$$

which, multiplied by the common denominator, becomes

$$4(x+3) + 4(x-3) = (x-3)(x+3)$$

which simplifies to

$$8x = x^2 - 9$$

so that x must satisfy the quadratic $x^2 - 8x - 9 = 0$. This can be seen, by either the quadratic formula or other methods, to have solutions $x = 9$ and $x = -1$. The answer $x = -1$ is clearly nonsense and results in nonsensical, negative time values, while the answer $x = 9$ works, and can in fact be varified to work: with a still-water speed of 9 miles per hour, the boat will travel at 6 miles per hour upstream, getting 4 miles in 40 minutes, and then travel at 12 miles per hour downstream, getting 4 miles in 20 minutes.