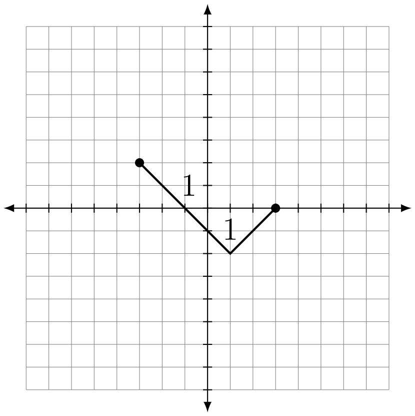
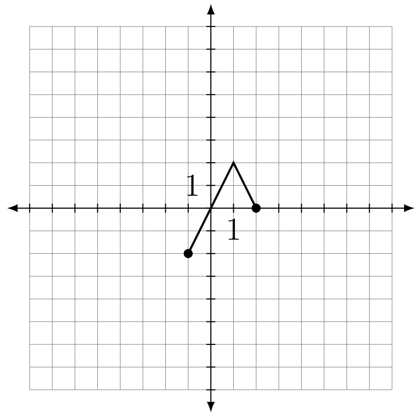
1. **(5 points)** *The graph on the left below depicts a function $f(x)$. Let $g(x) = -f(2x-1)$ and sketch the graph of $g(x)$ on the right.*





$f(x-1)$ is a shift of the graph of $f(x)$ one unit to the right; $f(2x-1)$ is furthermore a horizontal compression by a factor of 2. Finally, negating it flips it over the x-axis. Thus in particular, we can find that the three salient points $(-3,2)$, $(1,-2)$, and $(3,0)$ are mapped to $(\frac{-3+1}{2},-2)$, $(\frac{1+1}{2},2)$ and $(\frac{3+1}{2},0)$ respectively.

2. **(5 points)** *Find the inverse of the function $h(x) = \sqrt[5]{\frac{1}{2-3x}}$.*

We want to find a formula for x in terms of y from the equation $y = \sqrt[5]{\frac{1}{2-3x}}$; we can accomplish this algebraically.

$$\begin{aligned} y &= \sqrt[5]{\frac{1}{2-3x}} \\ y^5 &= \frac{1}{2-3x} \\ \frac{1}{y^5} &= 2-3x \\ 3x &= 2 - \frac{1}{y^5} \\ x &= \frac{2-\frac{1}{y^5}}{3} = \frac{2}{3} - \frac{1}{3y^5} \end{aligned}$$

so $h^{-1}(y) = \frac{2}{3} - \frac{1}{3y^5}$.

3. **(10 points)** *Given $f(x) = \frac{x+1}{x^2-4}$ and $g(x) = \sqrt{x}$, answer the following questions. Formulas need not be simplified.*

   (a) **(2 points)** *Give formulas for $(f+g)(x)$ and $\frac{g}{f}(x)$, labeling which is which.*

$$(f+g)(x) = f(x) + g(x) = \frac{x+1}{x^2-4} + \sqrt{x}$$
$$\frac{g}{f}(x) = \frac{g(x)}{f(x)} = \frac{\sqrt{x}}{\frac{x+1}{x^2-4}}$$

The latter can be simplified into $\frac{\sqrt{x}(x^2-4)}{x+1}$, if desired.

(b) **(3 points)** *Give formulas for $f(g(x))$ and $g(f(x))$, labeling which is which.*

$$f(g(x)) = f(\sqrt{x}) = \frac{\sqrt{x}+1}{(\sqrt{x})^2-4}$$

$$g(f(x)) = g\left(\frac{x+1}{x^2-4}\right) = \sqrt{\frac{x+1}{x^2-4}}$$

(c) **(5 points)** *Compute the domains of $(fg)(x)$ and $\frac{f}{g}x$, labeling which is which.*

It is easy to see that the domain of f excludes cases where the denominator $x^2 - 4 = 0$, so the domain consists of those points where $x \neq \pm 2$. The domain of g contains only the non-negative numbers (i.e., those points where $x \geq 0$).

The domain of fg is the overlap of these two regions, i.e., where $x \geq 0$ and $x \neq \pm 2$. We might write this as the pair of conditions $x \geq 0$ and $x \neq 2$, or the interval form $[0, 2) \cup (2, \infty)$ (we need not worry about $x = -2$, because it is already forbidden by the requirement $x \geq 0$).

The domain of $\frac{g}{f}$ is similar except we must also exclude the values of x where $f(x) = 0$. This is specifically the value $x = 0$, so our conditions become $x > 0$ and $x \neq 2$, or the interval $(0, 2) \cup (2, \infty)$.