1. **(11 points)** *Answer the following questions about the rational function* $f(x) = \frac{(2x+1)(x+2)}{(x+2)^2(x-3)}$.

   (a) **(3 point)** *What is the domain of this function?*
   Since the denominator is zero when $x+2=0$ or $x-3=0$, we must exclude -2 and 3 from the domain. We could state this as $x \neq -2$ and $x \neq 3$, or we could describe it in interval notation as $(-\infty, -2) \cup (-2, 3) \cup (3, \infty)$.

   (b) **(1 point)** *Calculate the* y*-intercept of this function, if it exists, or explicitly assert that it does not exist.*
   $f(0) = \frac{1 \cdot 2}{2^2(-3)} = \frac{-1}{6}$, so the y-intercept is at $(0, \frac{-1}{6})$.

   (c) **(3 point)** *What if any* x*-intercepts does this function have? If it has none, explicitly say so.*
   The numerator $(2x+1)(x+2)$ is zero when $x = \frac{-1}{2}$ or $x = -2$; the latter zero of the numerator is outside the domain of the function, so it is not a zero of the function as a whole. Thus, our only x-intercept is at $(\frac{-1}{2}, 0)$.

   (d) **(4 points)** *Find the horizontal, vertical, and slant asymptotes of this function. For each asymptote you identify, indicate which type it is, and give its equation. If the function has no asymptotes, explicitly say so.*
   The function resembles $\frac{2x^2}{x^3} = \frac{2}{x}$ in the long term; as x becomes very large in magnitude, this dwindles to zero, so there is a horizontal asymptote at $y = 0$. Because the numerator is nonzero and the denominator zero at $x = 3$, there is a vertical asymptote there. The behavior at $x = -2$, however, requires cancellation to determine: $f(x)$ is identical to the simplified expression $\frac{2x+1}{(x+2)(x-3)}$, which has an asymptote at $x = -2$, so $f(x)$ also has such an asymptote.

2. **(4 points)** *For which values of* x *is* $x^2 + 3x > 2x^2 + 2$*? Give your result in interval form.*

   We rearrange the inequality to be $0 > x^2 - 3x + 2$, the right side of which is factorable: $0 > (x-2)(x-1)$. We thus wish to find where $(x-2)(x-1)$ is negative. Knowing where it is zero, and knowing that in the long term it is positive, we may deduce that it is negative only on the interval $(1, 2)$.

3. **(5 points)** *For which values of* t *is* $\frac{t}{t+2} \leq \frac{t+4}{t}$*? Give your result in interval form.*

   We rearrange the inequality to be $\frac{t}{t+2} - \frac{t+4}{t} \leq 0$. We may find a common denominator on the left side:

$$\frac{t^2}{t(t+2)} - \frac{(t+4)(t+2)}{t(t+2)} \leq 0$$
$$\frac{t^2 - (t^2 + 6t + 8)}{t(t+2)} \leq 0$$
$$\frac{-6t - 8}{t(t+2)} \leq 0$$

   We are thus interested in the sign of the rational expression $\frac{-6t-8}{t(t+2)}$. Two points, $t = 0$ and $t = -2$, are outside its domain entirely. At $t = \frac{8}{-6} = \frac{-4}{3}$ it is zero. At all three of these points its sign will change. We note that for large t it will be negative, so we see that it is nonpositive, as desired, on the interval $(-2, \frac{4}{3}] \cup (0, \infty)$.