1. **(9 points)** *The radioactive substance quadium has a half-life of 17 days. Answer the following questions about a 75-gram sample of quadium.*

   (a) **(5 points)** *Write a function $f(t)$ representing the quantity of quadium left in the 75-gram sample after t days.*

   Since every seventeen days the sample size halves, we multiply the sample size by $\frac{1}{2}$ once for every seventeen days that have passed; in other words, we multiply by $\left(\frac{1}{2}\right)^{t/17}$. Thus our function for the sample size is $f(t) = 75\left(\frac{1}{2}\right)^{t/17}$.

   (b) **(5 points)** *Using your above formula, how many days will it take the 75-gram sample to decay down to contain only 15 grams of quadium?*

   For fun, let's start with a back-of-the-envelope estimation—which is emphatically not the way to come up with a precise answer to this question but is a nice skill. 15 is exactly one-fifth of 75; that is to say, it's less than a fourth (but not by much), and considerably more than an eighth. The sample will become a fourth of its original size in two half-lives (i.e., 34 days), and an eighth in three half-lives, which is 51 days. We're certain the right result is between those, probably closer to the former. An educated guess might be in the range of 36–40 days.

   But let's not make an educated guess; let's find the exact value! We want to know when $15 = 75\left(\frac{1}{2}\right)^{t/17}$, so let's solve for t in that equation:

$$\begin{aligned} 15 &= 75\left(\tfrac{1}{2}\right)^{t/17} \\ \tfrac{1}{5} = \tfrac{15}{75} &= \left(\tfrac{1}{2}\right)^{t/17} \\ \ln\tfrac{1}{5} &= \ln\left(\tfrac{1}{2}\right)^{t/17} \\ \ln\tfrac{1}{5} &= \tfrac{t}{17}\ln\tfrac{1}{2} \\ \frac{17\ln\frac{1}{5}}{\ln\frac{1}{2}} &= t \end{aligned}$$

   You could alternatively have taken a base-$\frac{1}{2}$ logarithm of both sides of the second line instead and used the change-of-base formula to get the same result.

   In actuality, $\frac{17\ln\frac{1}{5}}{\ln\frac{1}{2}} \approx 39.47$, so our estimate above was not too bad!

2. **(5 points)** *Simplify* $\log_4 18 + \log_4 100 - 2\log_4 15$*, writing your answer as a rational number.*

   Using logarithm laws:

$$\log_4 18 + \log_4 100 - 2\log_4 15 = \log_4 18 + \log_4 100 - \log_4(15^2) = \log_4 \frac{18\cdot 100}{15^2} = \log_4 8$$

   Note that 8 is not a simple exponent of 4, but it is a simple exopnent of 2, which is 4's square root, so $8 = 2^3 = (4^{1/2})^3 = 4^{3/2}$; thus $\log_4 8 = \frac{3}{2}$.

3. **(5 points)** *What is the domain of the function* $f(x) = \frac{x^2-3}{\log_{10}(4x-2)}$*?*

   There are two requirements here: the parameter of the logarithm must be positive, and the denominator of the fraction must be nonzero. Thus $4x-2>0$ and $\log_{10}(4x-2) \neq 0$. The first condition simplifies easily to $x > \frac{1}{2}$, while the second is $4x-2 \neq 10^0 = 1$, so $x \neq \frac{3}{4}$. Thus, the domain is $(\frac{1}{2}, \frac{3}{4}) \cup (\frac{3}{4}, \infty)$.