1. **(8 points)** *Perform the following tasks related to the trigonometry associated with* $t = \frac{-29\pi}{6}$.

   (a) **(2 points)** *What is the reference number of* $t = \frac{-29\pi}{6}$*?*
   Since $\frac{-29\pi}{6}$ is very nearly -5π, the reference number is $\left|-5\pi - \frac{-29\pi}{6}\right| = \frac{\pi}{6}$.

   (b) **(1 point)** *Which quadrant is the terminal point determined by* $t = \frac{-29\pi}{6}$ *in?*
   $\frac{-29\pi}{6}$ is very slightly more than -5π, so since -5π corresponds to a point on the negative y-axis, $\frac{-29\pi}{6}$ corresponds to a point slightly counterclockwise of the negative y axis, in the third quadrant.

   (c) **(1 point)** *What are the coordinates of the terminal point determined by* $t = \frac{-29\pi}{6}$*?*
   The aforementioned reference number $\hat{t} = \frac{\pi}{6}$ corresponds to the terminal point $\left(\pm\frac{\sqrt{3}}{2}, \pm\frac{1}{2}\right)$; since the terminal point is in the thrid quadrant, both of these are negative, for a terminal point of $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$.

   (d) **(1 point)** *What is* $\sin(\frac{-29\pi}{6})$*?*
   The sine of a value is the y-coordinate of its associated terminal point, and thus is $-\frac{1}{2}$ here.

   (e) **(1 point)** *What is* $\cos(\frac{-29\pi}{6})$*?*
   The cosine of a value is the x-coordinate of its associated terminal point, and thus is $-\frac{\sqrt{3}}{2}$ here.

   (f) **(2 point)** *What is* $\csc(\frac{-29\pi}{6})$*?*
   The cosecant of a value is the reciprocal of its sine, and thus is $\frac{1}{-1/2} = -2$ here.

2. **(4 points)** *Calculate* $\cot\frac{51\pi}{4}$.
   $\frac{51\pi}{4}$ describes a point with reference number $\frac{\pi}{4}$ in the second quadrant, so the associated terminal point is $\left(\frac{-\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ since the cotangent is the ratio of the x-coordinate to the y-coordinate, this cotangent is -1.

3. **(4 points)** *Find the amplitude and period of the function* $f(x) = -5\cos(3x)$. *Label which is which.*
   The amplitude is $|-5| = 5$, while the period is $\frac{2\pi}{|3|} = \frac{2}{3}\pi$.

4. **(4 points)** *The point* P *is on the unit circle in the third quadrant, and has* x*-coordinate* $\frac{-1}{5}$. *What is its* y*-coordinate?*
   A point on the unit circle must satisfy $x^2 + y^2 = 1$, and in this case $x = \frac{-1}{5}$, so

$$\begin{aligned}\left(\tfrac{-1}{5}\right)^2 + y^2 &= 1\\ \tfrac{1}{25} + y^2 &= 1\\ y^2 &= \tfrac{24}{25}\\ y &= \pm\sqrt{\tfrac{24}{25}} = \pm\tfrac{2}{5}\sqrt{6}\end{aligned}$$

   and since the point P is in the third quadrant, both of its coordinates are negative, so $y = -\frac{2}{5}\sqrt{6}$.

5. **(2 point bonus)** *A trigonometric function evaluation we did not see in class is that* $\cos\frac{2\pi}{5} = \frac{\sqrt{5}-1}{4}$. *Using that knowledge, determine the value of* $\csc\frac{-33\pi}{5}$ *on the back of this page.*

   Note that $\frac{-33\pi}{5}$ has reference number $\frac{2\pi}{5}$ and describes a point in the third quadrant. Thus $\csc\frac{-33\pi}{5} = \frac{-1}{\sin 2\pi 5}$.

   Since we know $\cos\frac{2\pi}{5}$, we may use the Pythagorean identity $\sin^2 x + \cos^2 x = 1$ to find $\sin\frac{2\pi}{5}$:

$$\begin{aligned}
\sin^2\frac{2\pi}{5} + \cos^2\frac{2\pi}{5} &= 1 \\
\sin^2\frac{2\pi}{5} + \left(\frac{\sqrt{5}-1}{4}\right)^2 &= 1 \\
\sin^2\frac{2\pi}{5} + \frac{5-2\sqrt{5}+1}{16} &= 1 \\
\sin^2\frac{2\pi}{5} &= \frac{10+2\sqrt{5}}{16} \\
\sin\frac{2\pi}{5} &= \pm\frac{\sqrt{10+2\sqrt{5}}}{4}
\end{aligned}$$

   Since $\frac{2\pi}{5}$ describes a point in the first quadrant, the positive solution is correct so

$$\csc\frac{-33\pi}{5} = \frac{-1}{\sin 2\pi 5} = \frac{-4}{\sqrt{10+2\sqrt{5}}}.$$

   If a rational denominator is desired, two multiplications and extensive simplification serve to accomplish that:

$$\begin{aligned}
\csc\frac{-33\pi}{5} &= \frac{-4}{\sqrt{10+2\sqrt{5}}} \cdot \frac{\sqrt{10+2\sqrt{5}}}{\sqrt{10+2\sqrt{5}}} \\
&= \frac{-4\sqrt{10+2\sqrt{5}}}{10+2\sqrt{5}} \cdot \frac{10-2\sqrt{5}}{10-2\sqrt{5}} \\
&= \frac{-4\sqrt{10+2\sqrt{5}}(10-2\sqrt{5})}{80} \\
&= \frac{-1\sqrt{(10+2\sqrt{5})(5-\sqrt{5})^2}}{10} \\
&= \frac{-\sqrt{40(5-\sqrt{5})}}{10} = \frac{-\sqrt{50-10\sqrt{5}}}{5}
\end{aligned}$$