

1. **(2 points)** Determine the value (in radians) of  $\arctan \frac{1}{\sqrt{3}}$ .

Since  $\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$  and  $\frac{\pi}{6}$  is within the interval  $(-\frac{\pi}{2}, \frac{\pi}{2})$ , we know that  $\arctan \frac{1}{\sqrt{3}} = \frac{\pi}{6}$ .

2. **(3 points)** Determine the value (in radians) of  $\arccos \frac{-1}{2}$ .

We know that the arc-cosine's range is  $[0, \pi]$ ; the section of this interval where the cosine is negative is  $(\frac{\pi}{2}, \pi]$ , and  $\frac{1}{2}$  is the cosine of the reference number  $\frac{\pi}{3}$ . We thus want a value in  $(\frac{\pi}{2}, \pi]$  with reference number  $\frac{\pi}{3}$ , and the number in question will be  $\frac{2\pi}{3}$ .

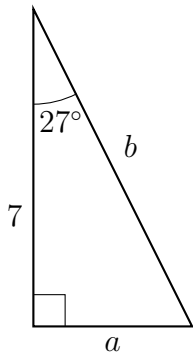
3. **(3 points)** What is the measure, in degrees, of an angle of  $\frac{7\pi}{4}$  radians?

There are 180 degrees in  $\pi$  radians, so that angle is  $\frac{7 \cdot 180}{4} = 315^\circ$ .

4. **(4 points)** Calculate  $\tan 210^\circ$ .

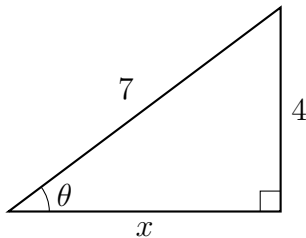
The reference number of  $210^\circ$  is  $|210^\circ - 180^\circ| = 30^\circ$ , so  $\tan 210^\circ = \pm \tan 30^\circ = \pm \frac{1}{\sqrt{3}}$ . Since  $210^\circ$  describes a point in the third quadrant, the desired tangent is specifically the positive possibility  $\frac{1}{\sqrt{3}}$ .

5. **(4 points)** Leaving your answer in terms of unsimplified trigonometric calculations on  $27^\circ$ , determine the values of the lengths  $a$  and  $b$  in the triangle below, indicating which is which.



We know the length of the adjacent side 7. Two relationships involving the adjacent side of this triangle are the cosine and tangent of  $27^\circ$ :  $\frac{7}{b} = \cos 27^\circ$  and  $\frac{a}{7} = \tan 27^\circ$ . We can then solve for the unknowns and get  $b = \frac{7}{\cos 27^\circ} = 7 \sec 27^\circ$  and  $a = 7 \tan 27^\circ$ .

6. **(4 points)** In the following triangle, determine the numerical value (fully simplified) of  $\tan \theta$ .



We might label the third side (as has been done above) with a length  $x$ . By the Pythagorean theorem,  $x^2 + 4^2 = 7^2$ , so  $x^2 = 49 - 16 = 33$  and, since lengths must be positive,  $x = \sqrt{33}$ . Since the tangent of an angle in a right triangle is the ratio of the opposite leg to the adjacent leg, we get  $\tan \theta = \frac{4}{\sqrt{33}} = \frac{4\sqrt{33}}{33}$ .