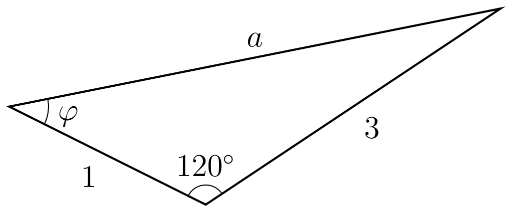
1. **(10 points)** *In the triangle drawn below, which is not to scale, find the requested measurements.*

   (a) **(5 points)** *The length a.* Fully simplify any trigonometric or inverse-trig functions used.

   Using the law of cosines, $a^2 = 1^2 + 3^2 - 2 \cdot 1 \cdot 3 \cdot \cos 120° = 13$, so $a = \sqrt{13}$.

   (b) **(5 points)** *The angle-measurement φ.* Do not simplify any trigonometric or inverse-trig functions whose values are unknown.

   We may use either the Law of Sines or the Law of Cosines. If we use the Law of Sines, we find that, using the value of a determined above,

   $$\frac{\sqrt{13}}{\sin 120°} = \frac{3}{\sin \varphi}$$

   so $\sin \varphi = \frac{3 \sin 120°}{\sqrt{13}} = \frac{3\sqrt{3}}{2\sqrt{13}}$. Thus, $\varphi = \arcsin\left(\frac{3\sqrt{3}}{2\sqrt{13}}\right)$.

   Using the Law of Cosines, we may consider φ to be the measure of the angle between sides of length a and 1 and opposite a side of length 3, which will thus satisfy

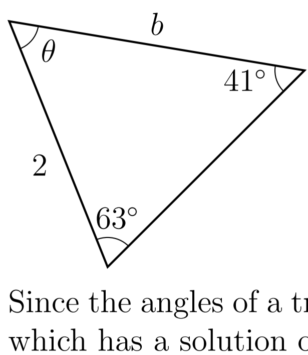
   $$\sqrt{13}^2 + 1^2 - 2\sqrt{13} \cdot 1 \cdot \cos \varphi = 3^2$$

   so $\cos \varphi = \frac{\sqrt{13}^2 + 1^2 - 3^2}{2\sqrt{13} \cdot 1} = \frac{5}{2\sqrt{13}}$ and so $\varphi = \arccos\left(\frac{5}{2\sqrt{13}}\right)$.

   In either case the actual evaluation of φ, which is not something you are remotely expected to do, comes out to approximately 46.1°.

2. **(7 points)** *In the triangle drawn below, which is not to scale, find the requested measurements.*

   (a) **(2 points)** *The angle-measurement θ.* Fully simplify any trigonometric or inverse-trig functions used.

   Since the angles of a triangle add up to 180°, it must be the case that $41° + 63° + \theta = 180°$, which has a solution of $\theta = 180° - 41° - 63° = 76°$.

   (b) **(5 points)** *The length b.* Do not simplify any trigonometric or inverse-trig functions whose values are unknown.

   Using the Law of Sines,

   $$\frac{b}{\sin 63°} = \frac{2}{\sin 41\circ}$$

   so $b = \frac{2 \sin 63°}{\sin 41°}$. This is not something which could reasonably be calculated by hand, but it is approximately equal to 2.72.

3. **(3 points)** *Write the trigonometric expression* $\sin x \tan x + \cos x$ *in as simple a form as possible.*

If we expand $\tan x$ as a ratio of sine and cosine and then convert the entire expression into one big fraction, we have

$$\sin x \tan x + \cos x = \sin x \frac{\sin x}{\cos x} + \cos x = \frac{\sin^2 x + \cos^2 x}{\cos x}$$

and using the Pythagorean identity, $\sin^2 x + \cos^2 x = 1$, so the above is further simplifiable to

$$\frac{1}{\cos x} = \sec x.$$