

1. The set of natural numbers  $\mathbb{N}$  is of course an infinite set; for each of the following conditions on a collection partitioning  $\mathbb{N}$ , either find such a collection or explain why it cannot exist.
  - (a) A collection  $\mathcal{A}$  partitioning  $\mathbb{N}$  such that  $\mathcal{A}$  is a finite collection, and every element of  $\mathcal{A}$  is a finite set.
  - (b) A collection  $\mathcal{B}$  partitioning  $\mathbb{N}$  such that  $\mathcal{B}$  is a finite collection, and every element of  $\mathcal{B}$  is an infinite set.
  - (c) A collection  $\mathcal{C}$  partitioning  $\mathbb{N}$  such that  $\mathcal{C}$  is an infinite collection, and every element of  $\mathcal{C}$  is a finite set.
  - (d) A collection  $\mathcal{D}$  partitioning  $\mathbb{N}$  such that  $\mathcal{D}$  is an infinite collection, and every element of  $\mathcal{D}$  is an infinite set.
  
2. Answer the following questions about Cartesian products.
  - (a) Explain why it is true that  $A \times (B \cup C)$  is the same set as  $(A \times B) \cup (A \times C)$ . Don't simply appeal to authority; explain why every element of the first set should be an element of the second and vice versa.
  - (b) Is  $(A \cup B) \times (C \cup D)$  the same set as  $(A \times C) \cup (B \times D)$ ? Why or why not?