1. (a) **(16 points)** Prove that there is no pair of sets A and B such that $A \cup B \subseteq A - B$ and $B \neq \emptyset$.

   (b) **(4 points)** When $B = \emptyset$, it is in fact the case that $A \cup B \subseteq A - B$. What aspect of your proof would fail to be valid when $B = \emptyset$?

2. **(10 points)** *Disprove* the following statement: for any natural numbers a and b, the sum $a + b$ is less than or equal to the product ab.

3. **(15 points)** Prove that for any integers n, a, b, and x, if $n \mid a$ and $n \mid b$, then $n \mid ax + b$.

4. **(20 points)** Prove that for any sets A, B, and C, if $A \subseteq B$, then $A - C \subseteq B - C$.

5. **(15 points)** Prove that for any integers a and b and natural number n, if $30a \not\equiv 45b \pmod{n}$, then $n \nmid 15$.

6. **(20 points)** Prove that for any natural number n, it is the case that $3 \mid n^3 - n$.

7. **(10-point bonus)** On the other side of this sheet, prove that for any natural number $p > 1$, if $2^p - 1$ is prime, then p is prime. (Note: the converse is not true, as $2^{11} - 1 = 23 \cdot 89$.)