

1. **(16 points)** Write out truth tables for each of the following statements (you may write them all in one truth table, if you wish):
  - (a)  $[\sim(P \vee Q)] \rightarrow \sim Q$ .
  - (b)  $(P \rightarrow Q) \leftrightarrow \sim P$ .
  - (c)  $(P \wedge \sim Q) \vee Q$ .
2. **(7 points)** Write the negation of the false statement “There is an even integer  $n$  such that  $n + 1$  is divisible by 4” as a quantified statement (using a universal or existential quantifier, either in words or in symbols).
3. **(11 points)** Determine the converse of the true statement “For a real number  $x$ , if  $x$  is positive, then  $x^2 + 5x$  is also positive.” Is the converse true? Either briefly justify your statement or provide a counterexample.
4. **(18 points)** Prove that for integers  $a$  and  $b$ , if both  $ab$  and  $a + b$  are even, then both  $a$  and  $b$  are even.
5. **(15 points)** Prove that for any integer  $n$ , the number  $5n^2 + 3n + 7$  is odd.
6. **(10 points)** Prove that if  $n$  is even, then  $3n - 5$  is odd.
7. **(16 points)** Let  $S = \{\emptyset, 2, 3, \{5, 7, 9\}, \{1, 2, 3, 4, \dots\}\}$ . For each of the following descriptions, either produce a set matching the description or explain briefly why such a set doesn't exist.
  - (a) A set  $A$  of 4 elements such that  $A \in S$ .
  - (b) A set  $B$  of 4 elements such that  $B \subseteq S$ .
  - (c) A set  $C$  of 4 elements, such that  $C \in \mathcal{P}(S)$ .
  - (d) A set  $D$  of 4 elements, such that  $D \subseteq \mathcal{P}(S)$ .