1. **(18 points)** *I want to borrow money to be repaid in five years; I predict that at that time I will be able to repay \$4500. My bank offers me a personal loan at 7.5% annual interest, compounded monthly.*

   (a) **(12 points)** *How much money could I safely borrow on these terms?*

   Here we know that this investment has a future value $F$ of 4500, a lifetime $t$ of 5 years, and an interest rate $r$ of 0.075 provided over twelve compounding periods per year. Our goal in this case is to determine the present value $P$ which would be associated with these other parameters. We shall plug all the information we have into the formula for present value:

   $$P = \frac{F}{\left(1+\frac{r}{n}\right)^{nt}} = \frac{4500}{\left(1+\frac{0.075}{12}\right)^{5\times 12}} \approx 3096.41$$

   so we could borrow $\boxed{\$3096.41.}$

   (b) **(6 points)** *What is the annual percentage rate of the above loan?*

   The APR would be

   $$\left(1+\frac{0.075}{12}\right)^{12} - 1 \approx 0.0776 = \boxed{7.76\%.}$$

2. **(18 points)** *If an investment of \$1000 grows in value to \$1500 in ten years, answer the following questions about its interest rate.*

   Here the present value $P$ is 1000, the future value $F$ is 1500, the lifetime $t$ is 10, and we wish to find the annual interest rate $r$ under each of the circumstances below.

   (a) *What annual rate of simple interest would produce this result?*

   Here $r = \frac{F-P}{Pt} = \frac{1500-1000}{1000\times 10} = \boxed{5\%}$.

   (b) *What annual rate of annually compounding interest would produce this result?*

   Here $r = \sqrt[t]{\frac{F}{P}} - 1 = \sqrt[10]{\frac{1500}{1000}} - 1 \approx \boxed{4.13\%.}$

3. **(16 points)** *Sara invests \$8000 in a high-yielding investment which returns 11% interest per year for six years. Determine the quantity of* interest *she earns in each of the following scenarios.*

   In all three cases that follow, the problem describes an instrument whose present value $P$ is 8000, annual interest rate $r$ is 0.11, and the number of years under consideration $t$ is 6; in all cases we seek the interest quantity $F - P$.

   (a) *The loan earns* simple interest*.*

   Simple interest is governed by the relationship $I = Prt$, so

   $$I = 8000 \times 0.11 \times 6 = 5280$$

   so she earns $\boxed{\$5280}$ in interest.

   (b) *The loan earns* annually compounding interest*.*

   Annually compounding interest is governed by the relationship $F = P(1+r)^t$, so

   $$F = 8000(1.11)^6 \approx 14963.32$$

   and thus the future value is \$14963.32, of which $\boxed{\$6963.32}$ is interest.

(c) *The loan earns* monthly compounding interest.

Periodically compounding interest is governed by the relationship $F = P\left(1+\frac{r}{n}\right)^{nt}$, where in this case $n = 12$ for monthly compounding, so

$$F = 8000(1+\tfrac{0.11}{12})^{6\times 12} \approx 15431.87$$

and thus her future value is \$15431.87, of which $\boxed{\$7431.87}$ is interest.

4. **(10 points)** *Dante has invested \$7000 into long-term treasury bonds which earn an annual interest rate of 3.2% compounded quarterly. How long will it take his investment to grow to \$10000?*

In this case what we seek is the lifetime $t$ (in years) or $m$ (in quarters) of an investment of known present and future value; we know the present value $P$ is 7000, and the desired future value $F$ is 10000. We also know the interest rate $r$ is 0.032; finally, the number of compounding periods per year is $n = 4$, since this compounds quarterly. Thus

$$m = \frac{\log \frac{F}{P}}{\log\left(1+\frac{r}{n}\right)} = \frac{\log \frac{10000}{7000}}{\log\left(1+\frac{0.032}{4}\right)} \approx 44.76$$

If left unrounded, the proper units are quarters, so this is $\boxed{44.76 \text{ quarters}}$, or we could round up to $\boxed{45 \text{ quarters}}$. Dividing by 5 would give the lifetime in years, so one could equally validly describe this as $\boxed{11.19 \text{ years}}$ if unrounded or $\boxed{11.25 \text{ years}}$ if rounded.

5. **(8 points)** *You have put \$2000 into a two-and-a-half year certificate of deposit which returns an annual interest rate of 3% compounding semiannually. How much will your investment be worth at the end of the CD's lifetime?*

We have a present value of $P = 2000$ growing at an annual rate of $r = 0.03$ for $t = 2.5$ years with semiannual ($n = 2$) compounding, and wish tro find the future value $F$, so:

$$F = P\left(1+\frac{r}{n}\right)^{nt} = 2000 \times \left(1+\frac{0.03}{2}\right)^{2.5\times 2} \approx 2154.57$$

for a final value of $\boxed{\$2154.57.}$

6. **(14 points)** *Answer the following questions about percentages and change.*

(a) **(7 points)** *A news report claims that the population of the town of Springcenter is 7.4% Native American, and further mentions that 45 Native Americans live in Springcenter. What is the total population of Springcenter?*

We know that 45 people make up 7.4% of the population. Thus the entire population is $\frac{45}{0.074} \approx 608$, so there are $\boxed{608}$ people in Springcenter. Note that 45 is, to one decimal place, actually 7.4% of 608. In fact, technically, assuming that 7.4% figure is rounded to one decimal place, Springcenter could have as many as 612 people, or as few as 605.

(b) **(7 points)** *A five-pound bag of flour costs \$1.60 at present, and the consumer price index is currently 147.98. In 1958 the consumer price index was 28.70. Based on this information, what would be the expected price for that same bag of flour in 1958?*

A bag of flour has a value equal to $\frac{1.60}{147.98} \approx 0.0108$ market baskets. THe value of this fraction of a market basket in 1958 would be $0.0108 \times 28.70 \approx 0.31$, so the bag of flour would have a time-shifted price of $\boxed{\$0.31}$ or $\boxed{31¢.}$

7. **(16 points)** *Javier borrows \$1000 for three years on a variable-rate loan which initially has an annual interest rate of 5% compounded quarterly. After a year the annual interest rate is increased to 7% (while compounding continues quarterly), where it remains for two years. How much will Javier need to pay back at the end of the loan's lifetime?*

   Since the interest rate will change, we may find it easiest to to calculate an "intermediary" future value. So the first four quarters of Javier's loan is described by a present value of \$1000 with an annual interest rate of 5% compounding each quarter. At the end of those four quarters, we could calculate the account balance to be

$$F = P\left(1+\frac{r}{n}\right)^m = 1000 \times \left(1+\frac{0.05}{4}\right)^4 \approx 1050.945337.$$

   Since this is an intermediary stage of our calculation, we want to keep it to as high a precision as possible. Then we can take this intermediary balance, and use it as an input to the new interest-bearing process which describes the remaining eight quarters. So, in order to figure out the final loan balance, we would take 1050.945337 as the present value of a loan which continues for eight quarters, accruing 7% annual interest compounding quarterly. We can use the same formula as above, with different parameters, to determine the eventual balance:

$$F = P\left(1+\frac{r}{n}\right)^m = 1050.945337 \times \left(1+\frac{0.07}{4}\right)^8 \approx 1207.41$$

   so his final repayment would be $\boxed{\$1207.41.}$

8. **(5 point bonus)** *In the last question, what is the effective APR on the loan taken as a whole?*

   The loan taken as a whole has a present value of \$1000, and a final value of \$1207.41, after three years, so its APR would be

$$r = \sqrt[3]{\frac{1207.41}{1000}} - 1 = \boxed{6.48\%.}$$