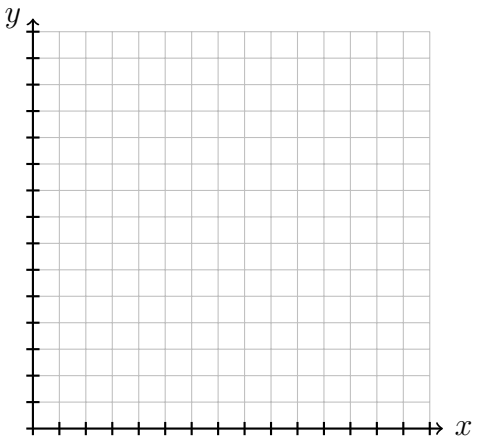
1. **(32 points)** Consider the linear programming problem in which we maximize the profit function $3x + 10y$ subject to the conditions

$$\begin{cases} 3x + 2y \leq 90 \\ x + 3y \leq 60 \\ y \leq 15 \\ x \geq 0, y \geq 0 \end{cases}$$

(a) **(10 points)** Sketch the constraint lines, labeling the intercepts, on the axes below, and shade the feasible region (indicate the scale on the axes).



(b) **(14 points)** Find the coordinates for each potential feasible profit-maximizing point in the graph above.

(c) **(8 points)** Find the value of the pair (x,y) maximizing the profit on the above graph.

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1	/ 32
2	/ 32
3	/ 36
Σ	/100

2. **(32 points)** You are producing children's bicycles, tricycles, and wagons in a workshop. Each bicycle uses two wheels and 3 feet of steel tubing, and can be sold for a profit of \$25. Tricycles each require three wheels, 4 feet of steel tubing, and a square foot of sheet metal; each tricycle yields \$40 in profit. Finally, wagons each require four wheels, use 2 feet of steel tubing each, and 5 square feet of sheet metal; these can be sold for a profit of \$35. You have 400 wheels, 500 feet of steel tubing, and 300 square feet of sheet metal.

   (a) **(14 points)** Produce a linear-programming formulation of this scenario, including constraints and a goal.

   (b) Could you produce 60 bicycles, 50 tricycles, and 40 wagons with the available materials? If not, why not, and if so, how much profit would you make and how much slack would remain in each constraint?

   (c) Could you produce 40 bicycles, 40 tricycles, and 50 wagons with the available materials? If not, why not, and if so, how much profit would you make and how much slack would remain in each constraint?

3. **(36 points)** Joanna has borrowed a 3D printer and will be using it to make fractal-shaped crafts to sell at a makerspace festival. She can program the printer to make a pair of Menger-sponge earrings with 4 grams of filament and 8 minutes of construction time; she can also make a Sierpinski-triangle trivet in 5 minutes using 10 grams of filament. Her earrings will sell for a profit of \$5 and the trivets for a profit of \$8. If she only has a single 540-gram spool of filament and has the printer for only 480 minutes (8 hours), then how many of each product should she make, and how much profit will that production plan create?

   The graph grid below is included for your use if desired.