

1. **(7 points)** You intend to buy a \$125,000 house with a 15% down payment and a 15-year mortgage with a 5.625% annual interest rate compounded monthly, with 1.5 discount points to be included in the loan. Answer the following three questions:

(a) What is the loan principal?

The 15% down payment is \$18,750, leaving \$106,250 to be borrowed. The inclusion of discount points in the loan requires a principal of $\frac{106250}{1-0.015} \approx 107868.02$, so the loan principal is $\boxed{\$107,868.02}$.

(b) What is the monthly payment amount?

Using the loan principal above as P , together with the loan characteristics $t = 15$, $n = 12$, and $r = 0.05625$, we find that the monthly payment is given by

$$A = \frac{Pi}{1 - (1 + i)^{-m}} = \frac{107868.02 \times \frac{0.05625}{12}}{1 - \left(1 + \frac{0.05625}{12}\right)^{-15 \times 12}} = 888.54$$

for a monthly payment of $\boxed{\$888.54}$.

(c) What is the total amount of interest paid over the lifetime of the loan (“finance charge”)?

The total repayment of $888.54 \times 180 = 159937.75$ includes both the principal and interest, so subtracting off the loan principal 107868.02 yields a total finance charge of $\boxed{\$52,069.73}$.

2. **(4 points)** You have a small-business loan of \$10,000 which you are repaying in equal quarterly installments over five years. The loan has an annual interest rate of 7%, compounded quarterly. What is the balance on the loan three years into its lifetime?

We wish to find the balance on a loan of principal $P = 10000$, with lifetime $m = 5 \times 4 = 20$ quarters, and with quarterly interest rate $i = \frac{0.07}{4} = 0.0175$, after the elapse of $m_0 = 3 \times 4 = 12$ quarters:

$$P_{m_0} = P \frac{1 - (1 + i)^{m_0 - m}}{1 - (1 + i)^{-m}} = 10000 \times \frac{1 - 1.0175^{-8}}{1 - 1.0175^{-20}} = 4420.16$$

for a remaining balance of $\boxed{\$4,420.16}$.

3. **(4 points)** By putting away \$250 each month in a savings account, you hope to at some point afford a \$9,000 used car. You have found a savings account that pays 3.2% annual interest compounded monthly for this purpose. How long will it take you to save up enough money for the car?

This is a long-term investment, and we wish to see when it will reach a desired future value $F = 9000$, using the parameters $A = 250$, $r = 0.032$, and $n = 12$:

$$m = \frac{\log\left(\frac{Fi}{A} + 1\right)}{\log(1 + i)} = \frac{\log\left(\frac{9000 \times \frac{0.032}{12}}{250} + 1\right)}{\log\left(1 + \frac{0.032}{12}\right)} = 34.421$$

so it would take 34.421 months (or 2.868 years), which you may round up to $\boxed{35 \text{ months}}$, which might equally well be called $\boxed{2.917 \text{ years}}$ or $\boxed{2 \text{ years and 11 months}}$. Note that this is only a very modest improvement on the 3 years it would take to accumulate that money simply stashing it under a mattress.