1. **(15 points)** *For an autumn bake sale, you're making apple-intensive baked goods. Each package of apple muffins you make uses 3 apples, a cup of sugar, and 2 cups of flour, and earns a profit of \$1.50. Each apple pie uses 10 apples, 2 cups of sugar, and 3 cups of flour, and earns a profit of \$3. Finally, each packet of apple cookies uses an apple, 3 cups of sugar, and a cup of flour and earns \$1 in profit. Your available supplies are 840 apples, 630 cups of sugar, and 450 cups of flour.*

   (a) **(8 points)** *Formulate a mathematical representation of the problem of maximizing profit in this scenario.*

   Let the variables x, y, and z represent the number of muffin-packs, pies, and cookie-packets made respectively. Then the profit in dollars gives the objective function $1.5x + 3y + z$. Finally, there are non-negativity constraints, as well as three constraints dictated by the limited quantities of apples, sugar, and flour. The problem thus becomes to $\boxed{\text{maximize } 1.5x + 3y + z}$ subject to the constraints:

$$\boxed{\begin{cases} 3x + 10y + z \leq 840 \\ x + 2y + 3z \leq 630 \\ 2x + 3y + z \leq 450 \\ x, y, z \geq 0 \end{cases}}$$

   (b) *Is it* feasible *to make 70 muffin-packs, 50 pies, and 120 cookie-packets? If so, what is the resulting profit, and what is the slack in each constraint? Make sure to show work or explain.*

   We test all of the resource constraints (the non-negativity constraints are clearly satisfied) to check to see if this production plan stays within our limitations:

$$\begin{aligned} 3 \times 70 + 10 \times 50 + 120 &= 830 \leq 840 \\ 70 + 2 \times 50 + 3 \times 120 &= 530 \leq 630 \\ 2 \times 70 + 3 \times 50 + 120 &= 410 \leq 450 \end{aligned}$$

   All constraints are satisfied, so this production plan is $\boxed{\text{feasible}}$ and it provides a profit of $\boxed{\$1.50 \times 70 + \$3 \times 50 + \$1 \times 120 = \$375.}$ It also has slack in all three resources, so that this plan will leave $\boxed{840 - 830 = 10 \text{ apples,}}$ $\boxed{630 - 530 = 100 \text{ cups of sugar,}}$ and $\boxed{450 - 410 = 40 \text{ cups of flour}}$ unused.

   (c) *Is it* feasible *to make 100 muffin-packs, 35 pies, and 180 cookie-packets? If so, what is the resulting profit, and what is the slack in each constraint? Make sure to show work or explain.*

   We test all of the resource constraints (the non-negativity constraints are clearly satisfied) to check to see if this production plan stays within our limitations:

$$\begin{aligned} 3 \times 100 + 10 \times 35 + 180 &= 830 \leq 840 \\ 100 + 2 \times 35 + 3 \times 180 &= 710 \not\leq 630 \\ 2 \times 100 + 3 \times 35 + 180 &= 485 \not\leq 450 \end{aligned}$$

   There are unsatisfied constraints (two of them, in fact), so this production plan is $\boxed{\text{not feasible}}$ (or equivalently, unfeasible or infeasible).