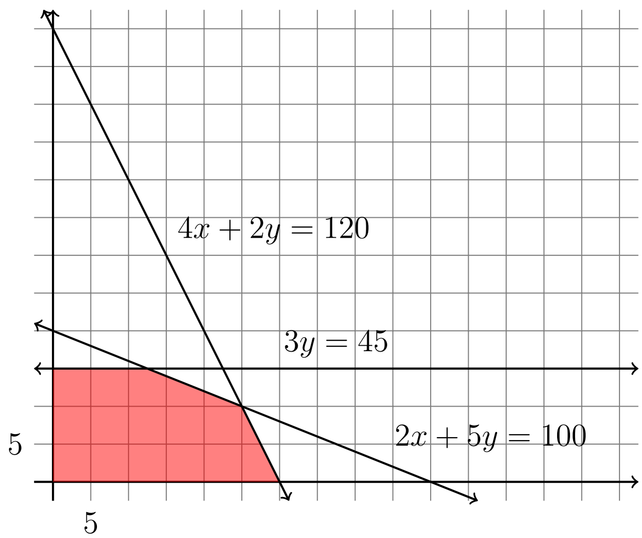
1. **(15 points)** *Answer the following questions about this linear-programming problem:*

$$\text{Maximize } 7x + 3y \text{ subject to the conditions } \begin{cases} 2x + 5y \le 100 \\ 3y \le 45 \\ 4x + 2y \le 120 \\ x, y \ge 0 \end{cases}$$

(a) **(5 points)** *Sketch the feasible region.*

The first constraint has intercepts of $(\frac{100}{2}, 0) = (50, 0)$ and $(0, \frac{100}{5}) = (0, 20)$, so we draw a line connecting those two points. The second constraint is a horizontal line with y-intecept of $(0, \frac{45}{3}) = (0, 15)$. The third constraint has intercepts of $(\frac{120}{4}, 0) = (30, 0)$ and $(0, \frac{120}{2}) = (0, 60)$, and we draw the line connecting them. Finally, the feasible region is the area below and to the left of these lines, and above and to the right of the axes, as shown below.



(b) **(7 points)** *Calculate the coordinates of every corner of the feasible region.*

Three are easy: $(0, 0)$ is the origin, $(30, 0)$ is the smallest x-intercept, and $(0, 15)$ is the smallest y-intercept. The other two are solutions of the systems of equations

$$\begin{cases} 2x + 5y = 100 \\ 3y = 45 \end{cases} \text{ and } \begin{cases} 2x + 5y = 100 \\ 4x + 2y = 120 \end{cases}$$

In the first system, we can calculate immediately from the second equation that $y = \frac{45}{3} = 15$, and then plug that into the first equation to get that $2x + 5 \cdot 15 = 100$, so $2x = 100 - 5 \cdot 15 = 25$, yielding $x = 12.5$. Thus, the coordinates of the first intersection point are $(12.5, 15)$.

In the second system, we might double the first equation to get $4x+10y = 200$; subtracting the second equation eliminates the x term, so that we get $(4x+10y)-(4x+2y) = 200-120$, and so $8y = 80$, from which $y = 10$. We can then substitute this back into either equation; if we use the first, we get $2x + 5 \cdot 10 = 100$, from which $2x = 100 - 5 \cdot 10 = 50$, so $x = 25$. Thus, this system produces the point with coordinates $(25, 10)$.

(c) **(3 points)** *Determine the values of x and y which maximize the objective function.*

From the work in the above section, we know the objective function is maximized at one of the corner points of the feasible region: $(0, 0)$, $(0, 15)$, $(12.5, 15)$, $(25, 10)$, or $(30, 0)$. We can reject $(0, 0)$ and $(0, 3)$ out of hand as inferior to $(12.5, 3)$, or we could include them in the test; it doesn't matter too much whether we do or not. If we test all five points, we get:

$$\begin{aligned} 7 \times 0 \quad &+ 3 \times 0 = 0 \\ 7 \times 0 \quad &+ 3 \times 15 = 45 \\ 7 \times 12.5 &+ 3 \times 15 = 132.5 \\ 7 \times 25 \quad &+ 3 \times 10 = 205 \\ 7 \times 30 \quad &+ 3 \times 0 = 210 \end{aligned}$$

So our maximizing values are $x = 30$ and $y = 0$, yielding an objective-function value of 210.