Section 1.1: Percentages

MATH 105: Contemporary Mathematics

University of Louisville

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Percentages in the everyday

On a good day of encountering the world around you, percentages come up all the time:

- "The poverty rate decreased to 13.5%, while median household incomes rose from $53,718 to $56,516—a 5.2% rise."
- "Save 30% or more on pre-owned devices!"
- "Tip 15% or more of the bill, based on the quality of service. If you receive exceptional service, 20-25% is customary."
- "Overcast with a 35 percent chance of rain, clearing in the afternoon.'

As a practical matter, percentages can describe anything you might want to divide up into groups: people, budgets, energy production and usage, even the fabric of reality!

Three major types of percentages

- **Parts of a whole**
  We use percentages to describe what portion of a whole belongs to a specific group.
  - "How much of the US population is Native American?" (1.2%)
  - "What percentage of global power generation is nuclear?" (10.6%)
- **Quantity of a change**
  A percentage cen describe how something changes: either over time, or because of some adjustment.
  - "Japan's population decreased by 0.8% from 2010 to 2015."
  - "Brewing for 60 minutes, use an extra 20% water for evaporation."
  - "Recently, the growth rate of the S&P 500 has been about 12% per year."
- **Likelihood of an event**
  A percentage can tell us how often something happens.
  - "You roll 6 on a pair of dice about 14% of the time."
  - "30% of the numbers we encounter routinely start with a '1', while only 5% start with a '9'."

Parts as a percentage

The governing relationship between a part, a whole, and the ratio between them is

$$\text{ratio} = \frac{\text{part}}{\text{whole}}$$

Using a calculator, the ratio is usually written as a decimal. "Per cent" literally means "out of every hundred", so

$$p\% = \frac{p}{100}$$

Or in other words, if we have a decimal we want to write as a fraction,

$$d = (100 \times d)\%$$

Calculating a part percentage: example

Kentucky demographics

As of the 2010 census, Kentucky had 4,339,367 people. 578,227 of them were 65 years old or older. What percentage is this?

We calculate the *ratio* between the *part* of interest (age 65+) and the *whole* (entire population):

$$\frac{578227}{4339367} \approx 0.1332514627$$

Now, to get a percentage, we multiply by 100:

$$0.1332514627 \times 100 \approx 13.3$$

so about **13.3%** of Kentucy's population is age 65 and up.

Time-saving tip

To multiply by 100, just shift the decimal point two places to the right.

Variations on part percentages

Recall that

$$\text{ratio} = \frac{\text{part}}{\text{whole}}.$$

This relationship among three quantities could be written in other ways, to solve for different quantities:

$$\text{part} = \text{whole} \times \text{ratio}$$

$$\text{whole} = \frac{\text{part}}{\text{ratio}}$$

These rephrasings allow you to solve different types of problems.

Examples with ratios

Energy usage in the US

Total energy consumption in the US in 2016 was 28,500,000 GWh, of which 39% was electrical power. How much electrical power was consumed in the US?

Note that here we know the *whole* (28.5 PWh total consumption) and the *ratio* (39%), but what we seek is the size of the part (electrical consumption). We calculate as follows:

$$39\% = \frac{39}{100} = 0.39$$

$$28500000 \times 0.39 = 11115000$$

so the US used about **11,115,000 GWh** of electricity in 2016.

Examples with ratios, cont'd

Demographics of South Korea

A news story says that South Korea has about 12,000,000 Buddhists, making up 22.8% of the population. What is the approximate population of South Korea?

In this case we know the *part* (12 million Buddhists) and the *ratio* (22.8%), and want to find the whole (total population). We calculate as follows:

$$22.8\% = \frac{22.8}{100} = 0.228$$

$$\frac{12000000}{0.228} \approx 52631579$$

so South Korea's population is about **52 million**.

Describing Change

When we describe change, we can put it in either *absolute* or *relative* terms.

For instance, the US population was 281,421,906 in 2000, and 308,745,538 in 2010. There are two different ways to describe what happened in those ten years:

Absolute change The US population grew by 27,323,632 people.

Relative change The US population grew by about 9.7%.

Relative change is calculated when we treat the absolute change as a *part* of the original value:

$$\frac{27323632}{281421906} \approx 0.097 = 9.7\%.$$

Change calculations

How do we calculate relationships among absolute change, percentage change, original value, and modified value?

$$\text{absolute change} = (\text{modified value}) - (\text{original value})$$

$$\text{relative change} = \frac{\text{absolute change}}{\text{original value}}$$

Or, to put it all in a single calcualation:

$$\text{relative change} = \frac{(\text{modified value}) - (\text{original value})}{\text{original value}}$$

Another way of looking at change

$$\text{relative change} = \frac{(\text{modified value}) - (\text{original value})}{\text{original value}}$$

Using some arithmetic, we can rewrite this as:

$$\text{relative change} = \frac{\text{modified value}}{\text{original value}} - 1$$

In other words, we can consider relative change to be theresult of thinking of the new value as a “part” of the original value’s “whole”, and then subtracting 100%.

Examples of change calculations

A year in the Dow Jones

The Dow Jones Industrial Average opened at 17,405 points in January 2016 and at 19,873 points in January 2017. What was its percentage change over 2016?

$$\text{absolute change} = 19873 - 17405 = 2468$$

$$\text{relative change} = \frac{2468}{17405} \approx 0.1418$$

so the DJIA grew **14.18%** over 2016.

Same problem, different calculation

A year in the Dow Jones

The Dow Jones Industrial Average opened at 17,405 points in January 2016 and at 19,873 points in January 2017. What was its percentage change over 2016?

$$\text{ratio} = \frac{19873}{17405} \approx 1.1418$$

so over 2016, the DJIA *grew to* 114.18% of what it was. 100% of that was there to begin with, so 14.18% of that is how much it *grew by*.

More change calculations

UofL enrollment number

UofL had 15,962 enrolled undergraduates in 2014–15, and 15,769 erolled undergraduates in 2015–16. What percentage change in enrollment occurred between these two years?

$$\text{absolute change} = 15769 - 15962 = -193$$

$$\text{relative change} = \frac{-193}{15962} \approx -0.0121$$

so enrollment at UofL went *down* by **1.21%**. Alternatively:

$$\frac{15769}{15962} \approx 0.9879$$

so enrollment *became* 98.79% of what it was, representing a decline of 1.21% (100%-98.79%).

Common changes to prices

Some specific terms are used to describe a change to the price of an item.

Markup The relative increase in price between the wholesaleprice at which it is bought, and the retail price at which it is sold.

Tax, tip, surcharge All of these are relative increases in cost, applied to the price.

Markdown, discount Terminology for a relative decrease in price, due to a sale or repricing.

Usage examples

Pricing of goods

A \$140 smartphone is being sold, in a special deal, for \$100. What is the discount rate?

As before, we can perform either of two calculations:

$$\frac{100 - 140}{140} \approx -0.286$$

$$\frac{100}{140} \approx 0.714$$

The top figure directly tells us that the phone is **discounted by 28.6%**. The second instead tells us that the phone costs 71.4% of what it used to; note then that $100\% - 71.4\% = 28.6\%$.

Usage examples, continued

Pricing of goods

The bulk price of high-quality chocolate is about \$3.50 per pound. The same chocolate is sold at retail for \$12 per pound. What is the markup?

We calculate either the markup directly or the ratio between the two prices:

$$\frac{12 - 3.50}{3.50} \approx 2.43$$

$$\frac{12}{3.50} \approx 3.43$$

The retail price *is* 343% of the wholesale price, but since 100% of that was already in the cost, the markup itself is only **243%**.

Variant calculations

$$\text{relative change} = \frac{(\text{modified value}) - (\text{original value})}{\text{original value}}$$

We can rearrange this formula to give ways to calculate the other quantities:

$$\text{modified value} = \text{original value} + \text{original value} \times \text{relative change}$$
$$\text{modified value} = \text{original value} \times (1 + \text{relative change})$$

$$\text{original value} = \frac{\text{modified value}}{1 + \text{relative change}}$$

Examples of variant calculations

Applying tax

A garden ornament costs \$13.50 and is subject to 6% tax. How much do you actually pay for it?

We could calculate the tax separately, and add it in:

$$\$13.50 \times 0.06 = \$0.81$$

$$\$13.50 + \$0.81 = \$14.31$$

Or, we could simply calculate the price-with-tax as 106% of the list price:

$$\$13.50 \times 1.06 = \$14.31$$

In either case, you will pay **\$14.31**.

Examples of variant calculations, continued

Reversing a surcharge

You paid \$47.24 in total for groceries which had a 8% delivery surcharge. How much did the groceries themselves cost?

Here we know that \$47.24 is 108% of the actual cost of the groceries, so we divide by 108% to get the original cost:

$$\frac{\$47.24}{1.08} \approx \$43.74$$

So the groceries cost **\$43.74** (with a \$3.50 delivery surcharge).

A trick question

Markup and markdown

A shirt costs \$5 to produce and is marked up by 20% for retail sale. It is then remaindered and discounted by 20%. What does it cost now?

A typical and sensible answer is \$5; this answer is incorrect!

$$\$5.00 \times 1.20 = \$6.00 \text{ (original retail price)}$$

$$\$6.00 \times 0.80 = \$4.80 \text{ (discounted retail price)}$$

so the shirt is actually now being sold for **\$4.80**.

Two different looks at the same issue

A political scenario

The 2004 plan to partially privatize Social Security was variously described as reducing Social Security taxes by either 2% or 32%. Why the discrepancy?

The proposal was going to reduce Social Security contribution from 6.2% of taxable income to 4.2%. This is clearly a difference of 2 *percentage points*, i.e. how much the SS contribution is *as a proportion of total income*.

On a hypothetical income of \$50,000, the old proposal took \$3,100; the new \$2,100. \$2,100 is about 67.7% of \$3,100, so the new proposal would involve a 32.2% *reduction in contribution size*.

In precise language: Social Security contribution was reduced by 32%, or by 2 *percentage points*.

Percentages of a net change

Another political scenario

"Women account for 92.3% of all jobs lost under Obama."
—Romney campaign, April 2012

That's an extraordinary claim! For each man who lost a job, did nearly 11 women really lose their job? Let's look at the job numbers.

	Jan. '09	Mar. '12	Net loss
Women only	66,122,000	65,439,000	683,000
Total	133,561,000	132,821,000	740,000

And $\frac{683000}{740000}$ is in fact about 92.3%.

But is that the whole story?

Percentages of a net change (continued)

Two factors explain that extraordinarily huge gender imbalance:

- Early, pre-2009 job loss hit male-dominated employment sectors.
- 2010–2012 job recovery also disproportionately occurred in male-dominated sectors.

To accentuate the absurdity, we could do the same calculation a mere month later, in February 2009:

	Feb. '09	Mar. '12	Net loss
Women only	65,923,000	65,439,000	484,000
Total	132,837,000	132,821,000	16,000

so in this time range, the share of job loss among women by the same calculation would be $\frac{484000}{16000} = 3025\%$!

The Romney campaign didn't use that number, because instead of looking dramatic, it was merely absurd.