Section 2.3: Amounts for Periodic Payments

MATH 105: Contemporary Mathematics

University of Louisville

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A more interesting question

Up until now we have asked three questions about installment plans:

- If we invest a certain amount over time, how much do we have at the end?
- If we know our repayment schedule, how much can we borrow?
- If we have a desired annuity payment, how much do we need to invest up front?

But these are the exact opposite of the things we're usually interested in!

- If we want to achieve a particular investment goal, how much do we need to periodically save?
- If we need to borrow a specific amount of money, how much will we need to pay each period?
- If we have a certain size of annuity deposit, how much income will it provide each period?

Rearranging our equations

Recall that we have two formulas, for two different types of financial instruments:

$$F = A\frac{(1+i)^m - 1}{i} \text{ (for long-term investments)}$$

$$P = A\frac{1-(1+i)^{-m}}{i} \text{ (for loans/annuities)}$$

But both of these will be very easy to solve for A, just by multiplying by an appropriate term:

$$\frac{Fi}{(1+i)^m - 1} = A \text{ (for long-term investments)}$$

$$\frac{Pi}{1-(1+i)^{-m}} = A \text{ (for loans/annuities)}$$

Some example questions

Investing for the future

If you have a 5%-annual-rate investment vehicle, compounding monthly, how large a deposit would you have to make each month to be a millionaire in 20 years?

Here our desired value of F is 1000000, $r = 0.05$, $t = 20$, and $n = 12$, and we want to know the value of A:

$$A = \frac{Fi}{(1+i)^m - 1} = \frac{1000000 \times \frac{0.05}{12}}{\left(1+\frac{0.05}{12}\right)^{20\times 12} - 1} \approx 2432.89$$

which is an awful lot to sock away each month! Maybe it's easier spread over 30 years?

$$A = \frac{Fi}{(1+i)^m - 1} = \frac{1000000 \times \frac{0.05}{12}}{\left(1+\frac{0.05}{12}\right)^{30\times 12} - 1} \approx 1201.55$$

More example questions

Avoiding work for a quarter year

You have \$400,000 to set up an annuity with. You've found a bank which will give you 4.5% annual interest compounding monthly on your investment. What monthly income will this annuity provide for the next 25 years?

Now P is 400000, $r = 0.045$, $t = 25$, and $n = 12$, and we want to know the value of A:

$$A = \frac{Pi}{1-(1+i)^{-m}} = \frac{400000 \times \frac{0.045}{12}}{1-\left(1+\frac{0.045}{12}\right)^{-25\times 12}} \approx 2223.33$$

which covers a fair number of expenses.

Even more example questions

Alternatives to rent-to-own

You want to pay for a 55-inch TV with a retail price of \$800 monthly over 19 months, so you put it on your credit card with an interest rate of 27.5% compounding monthly. How much should you pay each month?

In this case P is 800, $r = 0.275$, $m = 19$, and $n = 12$, and we want to know the value of A:

$$A = \frac{Pi}{1-(1+i)^{-m}} = \frac{800 \times \frac{0.275}{12}}{1-\left(1+\frac{0.275}{12}\right)^{-19}} \approx 52.41$$

which is a better payment than real rent-to-own will give you.

An illuminating contrast

Mortgages for a house

You need a \$100,000 mortgage to buy a house, and can get a 4.125% rate. What are the advantages and disadvantages of 30-year and 15-year (monthly-payment, and monthly-compounding) loans?

Here P is 100000, $r = 0.04125$, and $n = 12$, and we want to contrast the results of choosing $t = 15$ versus $t = 30$:

$$A_{15} = \frac{Pi}{1-(1+i)^{-m}} = \frac{100000 \times \frac{0.04125}{12}}{1-\left(1+\frac{0.04125}{12}\right)^{-12\times 15}} \approx 745.97$$

$$A_{30} = \frac{Pi}{1-(1+i)^{-m}} = \frac{100000 \times \frac{0.04125}{12}}{1-\left(1+\frac{0.04125}{12}\right)^{-12\times 30}} \approx 484.65$$

so a 30-year loan is a lot more affordable.

So 30-year loans are better?

If the monthly payment for a 30-year loan is \$484.65 and a 15-year loan is \$745.97, why would anyone ever want a 15-year loan?

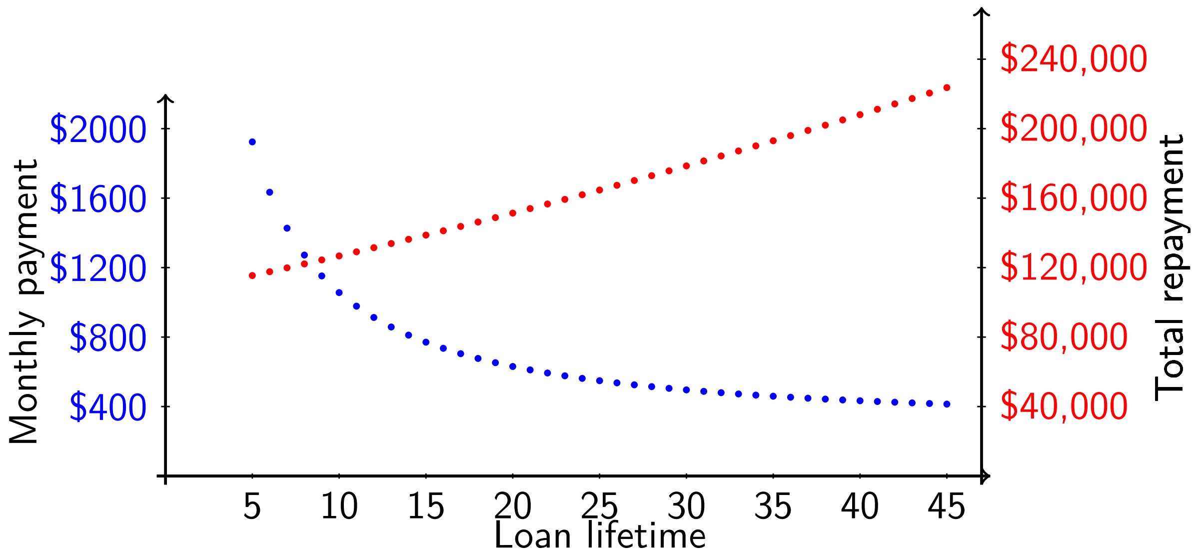
A 30-year loan is cheaper short-term, but more expensive *overall*:

$$484.65 \times 12 \times 30 = 174474.00$$

$$745.97 \times 12 \times 15 = 134274.60$$

In general, longer loans require smaller periodic payment, larger overall payment:

Hypothetical behavior of a \$100,000 4.125% loan



How to crash the economy

If we let t get very large, we will end up with the loan repayment/annuity payment formula:

$$A = \frac{Pi}{1-(1+i)^{-m}} \to \frac{Pi}{1-0} = Pi$$

so each period you pay or receive *only* interest.

These are "perpetual annuities" or "interest-only" loans.

The latter of these, together with the even more disasterous "negative-amortization" loan, played a large part in the mortgage catastrophes last decade.