Section 2.5: Mortgages

MATH 105: Contemporary Mathematics

University of Louisville

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## Mortgages as a type of amortized loan

We've already talked about mortgages a fair bit, in the context of "long-term loans". However, they also have some specialized language (some of which is shared with other types of loans).

These terms modify certain aspects of the loan, but it's important to remember that for the most part, mortages are entirely governed by the fundamental equations of amortized loans.

$$P = A \times \frac{1 - (1 + i)^{-m}}{i}$$

$$P_{m_0} = P \times \frac{1 - (1 + i)^{m_0 - m}}{1 - (1 + i)^{-m}}$$

# Special terms associated with mortgages

Down payment
A partial payment for the property in cash made up-front, which reduces the value of the principal to less than the value of the property.

Closing costs
A payment made up front (usually in cash, rarely added to the principal) covering the costs in finalizing the loan.

Discount points
A payment made up front (often added to the principal, less often paid in cash) to reduce the interest rate.

Refinancing
Ending a loan prematurely and moving its remaining balance into a new loan.

# Down payment

A down payment provides insurance for your loan financer because it usually guarantees your property is worth more than the balance on the loan.

The down payment is given as a *percentage* of the purchase price. Most loans require at least 3%, FHA loans require 3.5%, and private loans typically range from 5% to 15%.

### An example of a down payment

If you want to buy a $120,000 house, your lender might want a 15% down payment. How much cash would you need up front, and what yould your loan principal be?

The down payment is $0.15 \times \$120000 = \$18000$; what remains is $\$120000 - \$18000 = \$102000$. Thus, you would need to pay $18,000 in cash, and would borrow $102,000.

# Closing costs

Closing costs are the expenses associated with originating the loan. They might include appraisal fees, credit-check fees, title search fees, payments to legal or financial professionals, etc. Typically they are independent of the size of the mortgage.

### An example of closing costs

In last slide's $120,000 house with a 15% down payment, closing costs might be $2000. What options do we have to pay this?

*Paid up front*, these fees would add to your down payment, so you would need $20,000 in total cash upfront, and put $102,000 on the loan principal.

*Rolled into the loan*, these fees would be added to the loan principal, so you would need $18,000 in total cash upfront, and put $104,000 on the loan principal.

# Discount points

Discount points, or "points" are a percentage of the loan paid to reduce your interest rate.

Calculated up-front these aren't very complicated:

### An example of discount points

Continuing our example so far, our $120,000 purchase with 15% down and $2000 closing costs might also have 0.5 points. If we pay them up front, what's our loan setup?

We already had $20,000 cash up front and $102,000 in loan principal.

0.5% of that loan principal is $510, so we add $510 to our up-front costs and need $20,510 up front.

# Discount points, continued

## A trickier example of discount points

Our $120,000 purchase with 15% down and $2000 closing costs might also have 0.5 points. If we roll them into the loan, what happens?

We can't just add the $510 from the last slide into the loan principal! Why? Because that $510 is now part of the loan principal, and we owe points on *it* too.

What we really want is a quantity $p$, so that $p$ is 0.5% of the loan and points together.

$$p = 0.005(p + 102000)$$

whose solution is $0.995p = \$510$, or $p \approx 512.56$.

So our up-front costs here would be only $20,000, and our loan principal $104,512.56.

In general, if the principal before points is $L$ and the points as a percentage is $p$, the new loan principal will be $\frac{L}{1-p}$.

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# An explosion of different possibilities

Depending on whether things are paid up-front of added to the loan, there is significant variation in how the costs are distributed.

## Four variations on the same question

What are the different possibilities for how a mortgage on a $120,000 property with 15% down, $2000 closing costs, and 0.5 points might be set up?

| Configuration | Up-front payment | Loan principal |
|---|---|---|
| CC upfront, points upfront | $20,510.00 | $102,000.00 |
| CC upfront, points in loan | $20,000.00 | $102,512.56 |
| CC in loan, points upfront | $18,520.00 | $104,000.00 |
| CC in loan, points in loan | $18,000.00 | $104,522.61 |

# A mortgage setup problem, fully explored

## Everything you need to know all at once

You are buying a \$90,000 house on an FHA loan with a down payment of 3.5% and closing costs paid upfront of \$2500. The mortgage you've worked out is a 30-year fixed-rate loan with a 3.75% annual interest rate and one point rolled into the loan. What is your upfront cost, your monthly cost, and total finance charge?

This is a lot of information! But we can start by looking at the breakdown into upfront costs and principal.

$$\text{Upfront cost} = \text{DP} + \text{CC} = 0.035 \times 90000 + 2500 = 5150$$

$$P = (\text{price} - \text{DP}) \times \text{points} = (90000 - 0.035 \times 90000) \times \frac{1}{0.99} \approx 87727.27$$

# Continuing our exploration

## Just the loan part of the mortgage

This house purchase involves a 30-year fixed-rate mortgage of \$87,727.27 with a 3.75% annual interest rate. What is your monthly cost and total finance charge?

Mortgages are monthly, so we can use $P = 87727.27$, $r = 0.0375$, $t = 30$, and $n = 12$ to find the monthly payment.

$$A = \frac{P\frac{r}{n}}{1 - \left(1 + \frac{r}{n}\right)^{-nt}} = \frac{87727.27 \times \frac{0.0375}{12}}{1 - \left(1 + \frac{0.0375}{12}\right)^{-12 \times 30}} \approx 406.28$$

Finally, to find the total finance charge, note that over the entire life of the loan we make 360 payments of this size for a total of $360 \times A \approx 146260.32$.

Of this, \$87,727 is the loan principal, so the total interest ("finance charge" is $146260.32 - 87727.27 = 58533.05$.

# Our loan in summary

## Everything you need to know all at once

You are buying a $90,000 house on an FHA loan with a down payment of 3.5% and closing costs paid upfront of $2500. The mortgage you've worked out is a 30-year fixed-rate loan with a 3.75% annual interest rate and one point rolled into the loan.

| | |
|---|---|
| Up-front payment | $5,150 |
| Monthly payment | $406.28 |
| Total finance charge | $58,533.05 |

# What is refinancing?

*Refinancing* is taking a loan, midway through its lifetime, and paying it off by taking out a new loan.

Refinancing a loan can change its lifetime and interest rate in a way that serves you better.

But refinancing also comes with closing costs and possible discount points!

# How refinancing would work

## The loan we saw earlier, with a twist

Consider our aforementioned 30-year fixed-rate mortgage of $87,727.27 with a 3.75% annual interest rate and monthly payment of $406.28. Five years into the mortgage, the bank offers us a refinance to 3.5% interest with 0.75 discount points and $1500 in closing costs rolled into the loan. What are the consequences of taking this option?

We start by determining what our loan looks like after five years.

$$P_{60 \text{ months}} = 87727.27 \times \frac{1 - \left(1 + \frac{0.0375}{12}\right)^{60-360}}{1 - \left(1 + \frac{0.0375}{12}\right)^{-360}} \approx 79022.37$$

We still have 300 payments of $406.28 to make, so the remaining finance charge is

$$300 \times 406.28 - 79022.37 = 42861.63$$

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# How refinancing would work, continued

## The loan we saw earlier, with a twist

We are considering borrowing $79,022.37 for 30 years at 3.5% interest with 0.75 points and $1500 in closing costs rolled into the loan. What are this loan's vital statistics?

Our loan principal will be adjusted by closing costs and points:

$$P = \frac{79022.37 + 1500}{0.9925} = 81130.85$$

We can then calculate the monthly payment:

$$A = \frac{P\frac{r}{n}}{1 - \left(1 + \frac{r}{n}\right)^{-nt}} = \frac{81130.85 \times \frac{0.035}{12}}{1 - \left(1 + \frac{0.035}{12}\right)^{-12 \times 30}} \approx 364.31$$

so our total payment is $364.31 \times 360 \approx 131152.96$
and finance charge $131152.96 - 81130.85 \approx 50022.11$.

# So, should we refinance?

Let's look at how our original loan and our refinanced loan differ:

|  | Original | Refi |
|---|---|---|
| Principal | $79,022.37 | $81,130.85 |
| Monthly cost | $406.28 | $364.31 |
| Remaining periods | 300 | 360 |
| Total to be paid | $121,883.60 | $131,152.96 |
| Finance charge | $42,861.23 | $50,022.11 |

A refinanced loan typically ends up with higher payments *in total*, but can reduce the burden on you in the short term by bringing down monthly costs.