

Section 4.1: The Mathematics of Democracy

MATH 105: Contemporary Mathematics

University of Louisville

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Democracy and its challenges

The idea of democracy is simple: decisions are made by gathering a consensus opinion from among the viewpoints of the citizens.

In practice, there are many difficulties in enacting this well, but two which are at the forefront of the *theory of social choice*:

- ▶ In a decision among several alternatives, how do we accurately build a *societal consensus* out of many individual opinions?
- ▶ In a very large society, how do we assign representatives to large groups of individuals so that the viewpoints of the representative body reflects those of the citizenry as a whole?

The first of these issues is *voting theory*.

The second is *apportionment theory*.

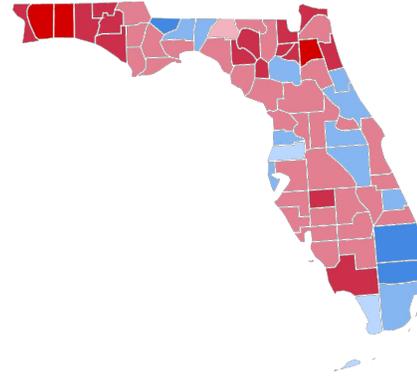
Both are places where the structures in place have arguably failed us.

Controversially skewed votes, part 1

The map to the right is a county-level breakdown of the presidential vote in Florida in 2000.

The significant candidates were Al Gore, George W. Bush, and Ralph Nader, with the following official vote totals:

Candidate	Vote totals	Percentage
Bush	2,912,790	48.847%
Gore	2,912,253	48.838%
Nader	97,488	1.635%



One problem is statistical: 537 votes falls under the threshold of ballot spoilage, miscount, and loss.

Another problem is civic: the 97,488 Nader voters almost certainly had a preference which was not taken into account.

Controversially skewed votes, part 2

Both the 2010 and 2014 Maine gubernatorial election were close races with several candidates.

2010 election			2014 election		
Candidate	Votes	%age	Candidate	Votes	%age
LePage (R)	218,065	37.6%	LePage (R)	294,533	48.2%
Cutler (I)	208,270	35.9%	Michaud (D)	265,125	43.4%
Mitchell (D)	109,387	18.8%	Cutler (I)	51,518	8.4%
Moody (I)	28,756	5.0%			

In both elections, a significant, vital percentage of the votes had no effect on a hotly contested outcome!

One solution would be to only allow two candidates. But another solution appeared on the ballot in 2016. . .

Maine Question 5

Official ballot summary

This initiated bill provides ranked-choice voting for the offices of United States Senator, United States Representative to Congress, Governor, State Senator and State Representative for elections held on or after January 1, 2018. Ranked-choice voting is a method of casting and tabulating votes in which voters rank candidates in order of preference, tabulation proceeds in rounds in which last-place candidates are defeated and the candidate with the most votes in the final round is elected.

This measure was passed by a vote of 383,660 (52%) to 354,351 (48%). But what does it do?

It establishes an alternative system for tabulating votes, which allows voters for candidates who have lost to nonetheless affect the outcome.

Three close elections in 229 years isn't bad, is it?

The issue arising from these cases is deeper than the mere problem of close elections.

In two of these three cases, the upset is blamed on a single third-party candidate.

These incidents sow distrust of the minor parties and encourage voters who want to “count” to support one of the two major parties.

Consequentially, the political spectrum is reduced to a cluster of candidates around two poles.

Our current political system thus promotes political polarization and discourages novel policy positions.

Anti-proportional factors in American representation

One of the axioms of democracy is that each citizen has an equal voice in the process of legislation.

In America, or any large nation, this can only be done through *legislative representation*, where a single person represents the viewpoint of a large cohort of citizens.

But these representatives are not distributed equally! Some of this is by design (the Senate), but even in the ostensibly proportional House the distribution is pretty lopsided:

State	Population	Representatives	Constituency size
Montana	989,415	1	989,415
Wyoming	563,626	1	563,626
Rhode Island	1,052,567	2	526,284
National average			708,377

An individual citizen of Montana has nearly half the voice in Congress as an individual citizen of Rhode Island or Montana!

What can we do?

Over the course of this unit, we're going to learn many things which can give us a better grasp on these issue, to become better participants in our democracy and to enact change wherever we can.

- ▶ What are the sensible ways to collect community preferences and build a consensus?
- ▶ What are the weaknesses and paradoxes in each system?
- ▶ Why is legislative rpresentation the way it is? What other methods are possible?
- ▶ How and why do apportionment methods fail?
- ▶ What pitfalls arise in apportionment beyond simple equality of district size?

Defining individual preferences

Most people’s views are not absolute single acceptable choices, but a preference order among multiple choices.

For example, in Florida in 2000, there were probably several different “preference orders” which were common among the voters:

- ▶ **Greens:** Nader > Gore > Bush
- ▶ **Left Democrats:** Gore > Nader > Bush
- ▶ **Center-right Democrats:** Gore > Bush > Nader
- ▶ **Republicans:** Bush > Gore > Nader

There are two other possible orderings, but they’d align with very atypical political positions.

So, if we want to take a voter’s *entire* viewpoint into consideration, we’ll consider their “ballot” to be not just a vote for their favorite, but an order among the candidates.

Simplifying voting results

If each voter’s ballot is an ordering among candidates, we’re likely to have a lot of individual, different ballots, like this:

	Individual ballots									
First choice	B	A	C	C	A	C	A	B	C	A
Second choice	C	B	A	B	B	B	B	A	A	C
Third choice	A	C	B	A	C	A	C	C	B	B

Building order out of chaos, we might collect identical ballots to build a *preference schedule*:

Number of votes	3	1	1	1	2	2
First choice	A	A	B	B	C	C
Second choice	B	C	A	C	A	B
Third choice	C	B	C	A	B	A

The central question of voting theory

Now that we know how to collect and collate public input, the real question comes to the forefront: if we have a lot of ballots, how do we turn that into a community-driven selection of a single candidate?

With only two candidates, the question is easy: either there is an authentic tie, or one candidate is preferred by more than half the population.

With more than two candidates, as we've seen, the question becomes more complicated. But we can start with the simplest and most familiar system.

Plurality vote rule

Give each candidate one point per voter who placed them as first preference. The candidate with the highest score is the *plurality vote winner*.

An example of plurality vote

Suppose Xavier, Yvonne, and Zachary are running for office and they gather the preference schedule of a group of 31 voters, as such:

Number of votes	7	2	12	9	1
First choice	X	X	Y	Z	Z
Second choice	Y	Z	X	X	Y
Third choice	Z	Y	Z	Y	X

In a plurality vote system, we would count up the total number of *first-place* votes only for each candidate:

- ▶ Xavier gets $7 + 2 = 9$ votes.
- ▶ Yvonne gets 12 votes.
- ▶ Zachary gets $9 + 1 = 10$ votes.

Thus, Yvonne would win this election.

This is the method used for most elections in the US; since we ignore second and third places, we don't even ask people about them.

Pros and cons of plurality vote

Plurality vote has a number of obvious benefits, mostly relating to its simplicity.

- ▶ Allows extremely simplified, easy to use ballots.
- ▶ Easily understood and very familiar to the public.
- ▶ Very easy to calculate.

However, as we've seen, it also comes with some dangers.

- ▶ Encourages two-party polarization.
- ▶ Disenfranchises unpopular viewpoints.

We might want to consider instead a system which recognizes and assigns value to the secondary preferences, to address these concerns.

A preference-weighting method



One potential approach to weighting preferences is to simply give points proportional to preference position: 1 point for last place, 2 for second-to-last, and so on.



This method was developed by the 13th-century hermetic philosopher Ramon Llull, but achieved prominence when proposed by the French Revolution theorist Jean-Charles de Borda.

It is thus now known as the *Borda count*.

Example of Borda count

Let's look at good old Xavier, Yvonne, and Zachary again:

Borda points	Number of votes	7	2	12	9	1
3	First choice	X	X	Y	Z	Z
2	Second choice	Y	Z	X	X	Y
1	Third choice	Z	Y	Z	Y	X

- ▶ Xavier gets $7 \times 3 + 2 \times 3 + 12 \times 2 + 9 \times 2 + 1 \times 1 = 70$ points.
- ▶ Yvonne gets $7 \times 2 + 2 \times 1 + 12 \times 3 + 9 \times 1 + 1 \times 2 = 63$ points.
- ▶ Zachary gets $7 \times 1 + 2 \times 2 + 12 \times 1 + 9 \times 3 + 1 \times 3 = 53$ points.

The Borda-count winner is Xavier, who did *not* win the plurality vote!

Pros and cons of the Borda count

Borda count is more complicated than plurality vote, but addresses some of its issues.

- ▶ Provides influence on the election even among voters with minority viewpoints.
- ▶ Provides an opportunity for “negative” voting by means of placing disliked candidates last.

Many democracies reject Borda count, however, for its disadvantages.

- ▶ Ballots are more complicated.
- ▶ Mediocre candidates perform unusually well.
- ▶ Still encourages strategic voting for likely-to-win candidates.

Nonetheless, Borda count is still used several places:

- ▶ Two constituencies in the Slovenian National Assembly
- ▶ Narauan Parliament and President of Kiribati (variant forms)
- ▶ Awarding of MLB MVP and Heisman Trophy
- ▶ Eurovision Song Contest (variant forms)