Section 4.1: Continuing Explorations of Borda Count
Section 4.2: Other voting methodologies

MATH 105: Contemporary Mathematics

University of Louisville

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The Borda Count and Mediocrity

Let us consider the following preference schedule among four candidates with 35 voters:

Number of votes	10	8	3	4	7	3
First choice	A	A	D	D	C	C
Second choice	B	B	C	B	B	D
Third choice	C	D	B	C	D	B
Fourth choice	D	C	A	A	A	A

In a plurality count A would be a runaway favorite: 18 votes to C's 10 and D's 7. In fact, A receives a *majority* (more than half) of the first-place votes.

In the Borda count, we have the following calculations:

- A: $10 \times 4 + 8 \times 4 + 3 \times 1 + 4 \times 1 + 7 \times 1 + 3 \times 1 = 89$.
- B: $10 \times 3 + 8 \times 3 + 3 \times 2 + 4 \times 3 + 7 \times 3 + 3 \times 2 = 99$.
- C: $10 \times 2 + 8 \times 1 + 3 \times 3 + 4 \times 2 + 7 \times 4 + 3 \times 4 = 85$.
- D: $10 \times 1 + 8 \times 2 + 3 \times 4 + 4 \times 4 + 7 \times 2 + 3 \times 3 = 77$.

The Borda Count and Mediocrity, continued

Number of votes	10	8	3	4	7	3
First choice	A	A	D	D	C	C
Second choice	B	B	C	B	B	D
Third choice	C	D	B	C	D	B
Fourth choice	D	C	A	A	A	A

B winning the Borda Count is troubling in two ways:

- B didn't get *any* first-place votes.
- A was the favorite of more than half the voters, but didn't win.

This motivates the following judgment of voting system quality:

The Majority Fairness Criterion

A voting system satisfies the *majority fairness criterion* if any candidate with a majority (more than half) of the first-place votes is guaranteed to win.

Majority Fairness, in more detail

A candidate with more than half the first-place votes is called a *majority candidate*.

An election with no majority candidate, or in which the majority candidate wins, does not tell you if the voting method satisfies the Majority Fairness Criterion.

But if an election where a majority candidate loses is *possible*, then the voting method does not satisfy the Majority Fairness Criterion.

Thus, we know the Borda Count *does not* satisfy the MFC.

Plurality voting, however, *does* satisfy the MFC.

A new method

This next method for voting is quite popular under a nubmer of different names. In the literature it's known as:

- Ranked choice voting (RCV) as seen in Maine Question 5
- Preferential voting
- Alternative-choice voting
- Transferable vote
- Instant-runoff voting (IRV)
- Plurality with elimination

What these names collectively tell us is that this is a method taking the entirety of voters' preference list into consideration, eliminating certain candidates and transferring their votes to other candidates.

An analogous idea: runoff voting

This voting system ("plurality with elimination" in the text) is a development from the idea of a *runoff election*, which is a twist on traditional plurality:

- As in a plurality vote, each voter selects their favorite candidate and the results are totaled up.
- If any candidate has a majority, they win the election.
- Otherwise, whichever candidate got the *fewest* first place votes will be eliminated and the election is run again.

The "instant" part of instant runoff is that if each voter submits a preference list on their ballot, the election doesn't need to be run again; the same ballots can be considered, reassigning votes for the eliminated candidate.

An IRV example

Let's look at the 31-voter election we saw last Thursday:

Number of votes	7	2	12	9	1
First choice	X	X	Y	Z	Z
Second choice	Y	Z	X	X	Y
Third choice	Z	Y	Z	Y	X

As deduced last Thursday, the plurality tabulation is 9 for X, 12 for Y, 10 for Z. Nobody has 16 votes, so nobody wins immediately, but X is *eliminated*:

# votes	7	2	12	9	1
1st choice	~~X~~	~~X~~	Y	Z	Z
2nd choice	Y	Z	~~X~~	~~X~~	Y
3rd choice	Z	Y	Z	Y	~~X~~

$\Rightarrow$

# votes	7	2	12	9	1
1st choice	Y	Z	Y	Z	Z
2nd choice	Z	Y	Z	Y	Y

Leaving the total at 19 for Y, 12 for Z; Y's 19 is a majority.

An interesting 3-system example

Here's an election with 21 voters, among alternatives A, B, and C:

Number of votes	10	2	9
First choice	A	B	C
Second choice	B	C	B
Third choice	C	A	A

In plurality vote, A wins with 10 votes.

In Borda count, B wins with a score of 44 to A and C's 41 each.

In IRV, the initial count has no majority, so B, with only 2 votes, is eliminated.

In the next round, A has 10 to C's 11, and C wins the instant-runoff vote.

This is thus an example of how these three methods of assessing community preference can get three different answers!

A rundown of IRV's pros and cons

IRV successfully addresses many of the concerns both of plurality and Borda count:

- A candidate supported by more than half the electorate will win.
- It is impossible to win without having *some* ardent supporters.
- Supporters of weak candidates can nonetheless influence the election with their vote.

Why isn't this the world's most popular voting system, then?

- Like Borda count, the ballot itself can be difficult for the public.
- It is subject to a few perverse behaviors (to be presented!)

That said, there are a lot of places which do use IRV.

- Almost all elections in Ireland
- Provincial elections in many parts of Canada
- Presidential elections in India
- Australian House of Representatives
- Several municipalities in the US

Where IRV fails

Let's look at the following 21-voter preference schedule and work out its IRV outcome:

Number of votes	8	2	5	6
First choice	A	B	B	C
Second choice	B	A	C	A
Third choice	C	C	A	B

Initial vote count is 8–7–6. No majority candidate, so C is eliminated:

Number of votes	8	2	5	6
First choice	A	B	B	~~C~~
Second choice	B	A	~~C~~	A
Third choice	~~C~~	~~C~~	A	B

And now A has 14 votes to B's 7; A wins. But how is this a problem?

Where IRV fails, continued

Let's take that preference schedule and imagine a slight tweak to it.

# votes	8	2	5	6
1st choice	A	B	B	C
2nd choice	B	A	C	A
3rd choice	C	C	A	B

$\Rightarrow$

# votes	8	2	5	6
1st choice	A	**A**	B	C
2nd choice	B	**B**	C	A
3rd choice	C	C	A	B

This modification has made A, who originally won the election, *even more popular*. Let's try running IRV on this new schedule.

First round is 10–5–6. B is out!

In the second round, A has 10 to C's 11. C wins!

The practical upshot: becoming *more popular* could cost a winner the election.

The problems with voting systems

Most voting systems have at least one way in which they behave perversely:

- Plurality vote completely ignores minority-candidate supporters.
- Borda count can be won by a mediocre candidate over a divisive one with majority support.
- An IRV winner's outcome can be harmed by increasing their support.

We named the second of these problems; likewise we can name the third.

Monotonicity Fairness Criterion

A voting method satisfies the *Monotonicity Fairness Criterion* if a winning candidate is guaranteed to continue to win if their position on some ballots is improved.