Show all your work, and explain why you use the arithmetic operations you use in reaching an answer.

1. **(20 points)** Answer the following questions about counting strings, sets, and partitions.

   (a) **(10 points)** Recall that the number of strings of 5 distinct letters chosen from an alphabet of 7 is $\frac{7!}{2!} = 2520$. We divide this by 120 to get the number of sets of 5 distinct letters (21).

   We also know that there are $7^5 = 16807$ strings of 5 freely chosen letters chosen from an alphabet of 7. Why can't we use a similar procedure to the one above to get the number of multisets of 5 letters from these 7 (of which there are 462, for reference). Describe the failure of this method in terms of the objects being investigated; the fact that 7^5 isn't divisible by 120 isn't enough justification!

   (b) **(10 points)** There are, as seen in class, 150 strings of length 5 using the letters A, B, and C, and using each letter at least once. There are 25 ways to partition the set $\{1, 2, 3, 4, 5\}$ into three subsets (possibly including empty sets). Describe how these two things being counted are related, and, specifically, how $150 = 6 \times 25$ is relevant to the counting.

2. **(10 points)** In some alien variant of poker played with a standard deck of cards, a *periflush* is a hand of five cards where two cards have the same number, and the other three cards are all of the same suit as each other and one of those two cards: for instance, 4♡ 4♣ J♡ 3♡ 7♡ is a periflush since the fours form a pair, and the other three cards together with one element of the pair are all of the same suit, while 9♠ 9♡ 3♡ Q♠ A♡ is not, since the non-pair cards are not all of the same suit. What is the probability that, drawing five cards at random, you draw a periflush? Recall that a standard poker deck consists of 52 distinct cards, with 13 different numbers and 4 different suits.

3. **(10 points)** Prove *combinatorially* that, for any integers k and n with $0 \leq k \leq n$,

$$k\binom{n}{k} = n\binom{n-1}{k-1}.$$

Musica est exercitium arithmeticae occultum nescientis se numerare animi.
—Gottfried Leibniz