Show all your work, and explain why you use the arithmetic operations you use in reaching an answer.

1. **(20 points)** Below we will be building a recursive formula to calculate the number of ways to build letter strings using each letter in our alphabet at least once. Let $S(n, k, r)$ represent the number of ways to build strings of k letters, drawn from an alphabet of n letters, if there are r letters which you are required to use at least once. For instance, the number of ways to build a 4-letter string using the letters A and B, if we are required to use at least one A, would be $S(2, 4, 1)$ (note that this is equal to 15).

   (a) **(7 points)** Explain why, for $k \geq r > 0$, it should be the case that

   $$S(n, k, r) = rS(n, k-1, r-1) + (n-r)S(n, k-1, r)$$

   (b) **(7 points)** Determine formulas (based on the definition of $S(n, k, r)$) for $S(n, k, 0)$ and $S(n, k, k+1)$.

   (c) **(6 points)** Use the results of the two previous sections to recursively calculate $S(3, 5, 3)$. Does this accord with information you've encountered elsewhere?

2. **(10 points)** I have selected a set of 30 different integers between 1 and 1000 (inclusive). Prove, without knowledge of which 30 integers I chose, that you can find seven different (but possibly overlapping) four-element subsets of my set whose elements all add up to the same number.

3. **(10 points)** If you have a list of n numbers, how many operations it take to determine whether there are numbers x, y, and z in the list such that $x + y = z$? Put your answer in "big-O" form and briefly explain your algorithm.

שתי אבנים בונות שני בתים: שלש אבנים בונות ששה בתים: ארבע אבנים ב ונות ארבעה ועשרים בתים: חמש אבנים בונות מאה ועשרים בתים: שש אבנים בונות ש בע מאות ועשרים בתים: שבע אבנים בונות חמשת אלפים וארבעים בתים: מכאן ו אילך צא וחשוב מה שאין הפה יכול לדבר ואין האוזן יכולה לשמוע

—ספר היצירה פרץ ד׳ משנה ט״ז

 due September 21, 2017