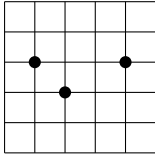


For full credit show all of your work (legibly!), unless otherwise specified. Answers need not be completely reduced unless otherwise stated — and for many questions, the calculation will be unpleasantly burdensome — and may be left in terms of sums, differences, products, quotients, factorials, and binomial coefficients.

1. **(10 points)** How many direct paths are there from the lower left corner to the upper right corner of the following grid which *pass through at least one of the marked points*?



2. **(15 points)** Show via a combinatorial proof that for $0 \leq k \leq n$,

$$\sum_{j=k}^n \binom{n}{j} \binom{j}{k} = \binom{n}{k} 2^{n-k}.$$

3. **(5 points)** What is the coefficient of x^2 in the expansion of $(3x + 4)^6$?
4. **(25 points)** A far-future dystopia is replacing everyone's names with 7-digit codes using only the digits 1, 2, 3, and 4.
- (a) **(5 points)** How many different "names" are possible under this scheme?
- (b) **(10 points)** Two people are said to be in the same artificial family if their names are anagrams of each other; for instance, citizens 1314221 and 2211134 are both in the same family. How many different families are there?
5. **(10 points)** Describe (with brief justification) in big-O notation the time taken by the following procedure for finding the dot products of two n -dimensional vectors \mathbf{a} and \mathbf{b} .
- Initialize $z \leftarrow 0$.
 - For each value of i from 1 to n :
 - Assign $z \leftarrow z + a_i b_i$.
 - Return the value of z and terminate.

6. **(15 points)** Galangal is a ginger-like rhizome used in Southeast Asian cuisine. Answer the following questions about anagrams of the word “GALANGAL”.

(a) **(5 points)** How many anagrams are there in total?

(b) **(10 points)** How many anagrams are there which do not contain either a double G or a double L (i.e. “GLAANLGA” is OK, “LANGGLAA” is not).