Turn the following true statements into propositions with explicitly described variables, premise, and conclusion, and then prove that proposition.

1. *The cube of any even integer is even.*

   Proposition 1. *For integer n, if n is even, then n^3 is even.*

   Proof. From our premise that n is even, we may determine that $n = 2k$ for some integer k. Thus $n^3 = (2k)^3 = 8k^3 = 2(4k^3)$. Since $4k^3$ is an integer, n^3 is definitionally even. □

2. *If the square of a rational number is not an integer, then the rational number itself is also not an integer.*

   Proposition 2. *For a rational number q, if $q^2 \notin \mathbb{Z}$, then $q \notin \mathbb{Z}$.*

   Proof. Let us instead prove the contrapositive, that if $q \in \mathbb{Z}$, then $q^2 \in \mathbb{Z}$. This is a triviality: since the product of any two integers is an integer, our new premise that q is an integer guarantees that $q \cdot q$ is also an integer. □

3. *Any non-negative real number plus one is positive.*

   Proposition 3. *For a real number x, if x is non-negative, then $x + 1$ is positive.*

   Proof. From our premise, we may assume that $x \geq 0$. Then $x + 1 \geq 1$. Since $1 > 0$, we then derive that $x + 1 > 0$. Thus, $x + 1$ is positive. □

4. *If the product of two integers is even, then at least one of the two factors in the product must be even.*

   Proposition 4. *For integers m and n, if mn is even, then either m is even or n is even.*

   Proof. Let us instead prove the contrapositive, that if neither m nor n is even, then mn is not even. Noting that noneven integers are odd, we may more concisely phrase the statement to be proven as: if m and n are both odd, then mn is odd. From our premises that m is odd and that n is odd, we may respectively deduce that $m = 2k+1$ for some $k \in \mathbb{Z}$ and that $n = 2\ell+1$ for some $\ell \in \mathbb{Z}$. Then

   $$mn = (2k+1)(2\ell+1) = 4k\ell + 2k + 2\ell + 1 = 2(2k\ell + k + \ell) + 1$$

   and since $2k\ell + k + \ell$ is an integer, the above demonstrates that mn is odd. □