1. **(20 points)** Prove that the equation $x^3 + 3x = 14$ has *exactly one* positive real solution.

2. **(20 points)** Prove that for any positive integer n, it is the case that

$$1 \cdot 2^1 + 2 \cdot 2^2 + 3 \cdot 2^3 + 4 \cdot 2^4 + \cdots + n \cdot 2^n = (n-1)2^{n+1} + 2$$

3. **(5 points)** *Disprove* the following statement: If x and y are real numbers such that $x < y$, then it is also the case that $x^2 < y^2$.

4. **(15 points)** Prove that for any integer n, if n is odd, then $8 \mid n^2 - 1$.

5. **(20 points)** Prove that there is no integer n such that $n \equiv 4 \pmod{6}$ and $n \equiv 2 \pmod{9}$.

6. (a) **(15 points)** Prove that for any sets A, B, and C, $(A \cup B) - C \subseteq A \cup (B - C)$.

   (b) **(5 points)** Demonstrate that for sets A, B, and C, it is *not* necessarily true that $(A \cup B) - C = A \cup (B - C)$.