1. **(5 points)** *Find the equation of the tangent line to the curve* $y = \frac{x^3-3x^2}{x-3}$ *at the point* $(4, 16)$.

   As written this problem has a foolishly simple solution, since the formula can be simplified to $y = x^2$, and so $\frac{dy}{dx} = 2x$ yielding a slope at the point $x = 4$ of $2 \cdot 4 = 8$ for the tangent line. If, however, you make use of the quotient rule, you can come to the same conclusion, albeit by a roundabout route:

   $$\frac{dy}{dx} = \frac{(x-3)\frac{d}{dx}(x^3-3x^2) - (x^3-3x^2)\frac{d}{dx}(x-3)}{(x-3)^2} = \frac{(x-3)(3x^2-6x) - (x^3-3x^2)\cdot 1}{(x-3)^2}$$

   which evaluates at $x = 4$ to $\frac{(4-3)(48-24)-(64-48)}{(4-3)^2} = 8$.

   In either case, once the slope of the tangent line is known to be 8, its equation can be found in point-slope form to be $y - 16 = 8(x-4)$ or in slope-intercept form as $y = 8x - 16$.

2. **(4 points)** *Determine* $\frac{d}{dx}\left(x^3 \sec x\right)$.

   Using the product rule:

   $$\frac{d}{dx}\left(x^3 \sec x\right) = \left(\frac{d}{dx}x^3\right)\sec x + x^3 \frac{d}{dx}\sec x = 3x^2 \sec x + x^3 \sec x \tan x = x^2 \sec x(3 + x \tan x).$$

   The final factorization in the above line is purely cosmetic and is not necessary to have a correct answer.

3. **(4 points)** *Determine* $\frac{d}{du}\frac{e^u}{u^2+\cot u}$.

   Using the quotient rule:

   $$\frac{d}{du}\frac{e^u}{u^2+\cot u} = \frac{(u^2+\cot u)\frac{d}{du}e^u - e^u \frac{d}{du}(u^2+\cot u)}{(u^2+\cot u)^2} = \frac{(u^2+\cot u)e^u - e^u(2u - \csc^2 u)}{(u^2+\cot u)^2}.$$

4. **(4 points)** *Determine* $\frac{d^2}{d\theta^2}\tan\theta$.

   We know $\frac{d}{d\theta}\tan\theta = \sec^2\theta = (\sec\theta)(\sec\theta)$. To calculate another derivative, we could use the product rule:

   $$\begin{aligned}\frac{d^2}{d\theta^2}\tan\theta &= \frac{d}{d\theta}\left(\sec\theta\sec\theta\right) = \left(\frac{d}{d\theta}\sec\theta\right)\sec\theta + \sec\theta\frac{d}{d\theta}\sec\theta \\ &= \sec\theta\tan\theta\sec\theta + \sec\theta\sec\theta\tan\theta = 2\sec^2\theta\tan\theta.\end{aligned}$$

5. **(3 points)** *Determine* $\frac{d^{27}}{dx^{27}}\left(2e^x - 4\cos x\right)$.

   The above by simple derivative rules will equal $2\frac{d^{27}}{dx^{27}}e^x - 4\frac{d^{27}}{dx^{27}}\cos x$. Differentiating e^x anynumber of times leaves it unchanged, while differeniating $\sin x$ or $\cos x$ moves it throuh a cycle of length 4; 24 of the derivatives will thus take $\cos x$ back to where it started, and the next three will transform it to $-\sin x$, then $-\cos x$, and finally to $\sin x$. Thus this derivative is $2e^x - 4\sin x$.