1. **(4 points)** *Find the equation of the tangent line to the curve $y = \sqrt{x^3 - 2x + 5}$ at the point $(2, 3)$.*

Using the chain rule with $u = x^3 - 2x + 5$, we have

$$\frac{dy}{dx} = \frac{d}{dx}\sqrt{u} = \frac{d}{du}u^{1/2}\frac{du}{dx} = \frac{3x^2 - 2}{2\sqrt{x^3 - 2x + 5}}$$

yielding a slope at the point $x = 2$ of $\frac{3 \cdot 2^2 - 2}{s\sqrt{2^3 - 2 \cdot 2 + 5}} = \frac{5}{3}$ for the tangent line.

Thus the equation of the tangent line can be found in point-slope form to be $y - 3 = \frac{5}{3}(x - 2)$ or in slope-intercept form as $y = \frac{5}{3}x - \frac{1}{3}$.

2. **(8 points)** *Determine a formula for $\frac{dy}{dx}$ by implicitly differentiating $\frac{x^2}{x+y} = y^3 - x$.*

There are multiple approaches here; for instance, pre-emtively multiplying by $x+y$ means using the product rule on the right side instead of the quotient rule on the left, but here is a naïve approach to finding $\frac{dy}{dx}$ via implicit differentiation.

$$\begin{aligned}
\frac{d}{dx}\frac{x^2}{x+y} &= \frac{d}{dx}(y^3 - x) \\
\frac{(x+y)\frac{d}{dx}(x^2) - x^2\frac{d}{dx}(x+y)}{(x+y)^2} &= \left(\frac{d}{dx}y^3\right) - 1 \\
\frac{(x+y)(2x) - x^2(1 + \frac{dy}{dx})}{(x+y)^2} &= \left(\frac{d}{dy}y^3\right)\frac{dy}{dx} - 1 \\
\frac{(x+y)(2x) - x^2 - x^2\frac{dy}{dx}}{(x+y)^2} &= 3y^2\frac{dy}{dx} - 1 \\
\frac{(x+y)(2x) - x^2}{(x+y)^2} + 1 &= 3y^2\frac{dy}{dx} + \frac{x^2}{(x+y)^2}\frac{dy}{dx} \\
\frac{\frac{(x+y)(2x)-x^2}{(x+y)^2} + 1}{3y^2 + \frac{x^2}{(x+y)^2}} &= \frac{dy}{dx} \\
\frac{(x+y)(2x) - x^2 + (x+y)^2}{3y^2(x+y)^2 + x^2} &= \frac{dy}{dx}
\end{aligned}$$

This expression is definitely simplifiable, and the last line is an (optional) simplification borne of multiplying both the numerator and denominator by $(x+y)^2$.

3. **(8 points)** *Calculate the following derivatives.*

(a) For $f(t) = 6t^2 \arcsin(t)$, find $f'(t)$.
Using the product rule,

$$f'(t) = \left(\frac{d}{dt}6t^2\right)\arcsin t + 6t^2\left(\frac{d}{dt}\arcsin t\right) = 12t\arcsin t + \frac{6t^2}{\sqrt{1 - t^2}}.$$

(b) Calculate $\frac{d}{dq}\arctan(3q + 1)$.
Using the chain rule with $u = 3q + 1$,

$$\frac{d}{dq}\arctan(3q+1) = \frac{d}{dq}\arctan(u) = \frac{du}{dq}\frac{d}{du}\arctan(u) = 3 \cdot \frac{1}{1 + u^2} = \frac{3}{1 + (3q+1)^2}.$$

(c) Calculate $\frac{d}{dx}\frac{\sin x}{\ln(x)}$.

Using the quotient rule,

$$\frac{d}{dx}\frac{\sin x}{\ln(x)} = \frac{\ln x \frac{d}{dx}\sin x - \sin x \frac{d}{dx}\ln x}{(\ln x)^2} = \frac{\ln x \cos x - \frac{\sin x}{x}}{(\ln x)^2}.$$