1. **(6 points)** *A spherical balloon is being filled by a compressor which moves 600 cubic centimeters of air (at the balloon's internal pressure) per second. The balloon currently has a radius of 50cm (i.e. a diameter of 1 meter). How quickly is the balloon's radius growing right now? For reference, the volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.*

   Since $V = \frac{4}{3}\pi r^3$ and $\frac{dV}{dt} = 600$, we may use implicit differentiation to determine that

$$\begin{aligned} \frac{d}{dt}V &= \frac{d}{dt}\frac{4}{3}\pi r^3 \\ \frac{dV}{dt} &= \frac{dr}{dt}\frac{d}{dr}\frac{4}{3}\pi r^3 \\ 600 &= \frac{dr}{dt}4\pi r^2 \\ \frac{600}{4\pi r^2} &= \frac{dr}{dt} \end{aligned}$$

   and since $r = 50$ currently, the rate of change of the radius is $\frac{600}{10000\pi} \approx 0.002$ centimeters per second.

2. **(9 points)** *You are using a radargun to track the speed of a car which is currently 0.8 miles to the west of your monitoring position on an east-west racetrack; for safety, you are standing 0.6 miles south of the track. Your radargun reports that the car is approaching you at 160 miles per hour.*

   (a) **(5 points)** *How quickly is the car actually moving?*

   Here y is a constant 0.6 and $\frac{ds}{dt} = -160$ from your radar measurement. Using the Pythagorean Theorem $x^2 + y^2 = s^2$, we may implicitly differentiate in hopes of finding $\frac{dx}{dt}$. Then, knowing that x is currently 0.8 and $s = \sqrt{x^2+y^2} = 1.0$, we implicitly differentiate the Pythagorean Theorem to get

$$\begin{aligned} \frac{d}{dt}(x^2+y^2) &= \frac{d}{dt}s^2 \\ \frac{dx}{dt}\frac{d}{dx}x^2 + \frac{dy}{dt}\frac{d}{dy}y^2 &= \frac{ds}{dt}\frac{d}{ds}s^2 \\ 2x\frac{dx}{dt} + 2y\frac{dy}{dt} &= 2s\frac{ds}{dt} \\ 2x\frac{dx}{dt} &= 2s\frac{ds}{dt} - 2y\frac{dy}{dt} \\ \frac{dx}{dt} &= \frac{s\frac{ds}{dt} - y\frac{dy}{dt}}{x} \end{aligned}$$

   Then $\frac{dx}{dt} = \frac{1(-160)-0.6\cdot 0}{0.8} = -200$, so the car is traveling at 200 miles per hour.

   (b) **(4 points)** *If you want to continue tracking the car, how quickly should you be swiveling the radar at this moment?*

   We proceed with the same knowledge as above, but also need to make use of a trigonometric measurement related to the angle your radargun makes with some fixed direction; e.g. we can let θ be the angle between your radargun and due north, and then try to calculate $\frac{d\theta}{dt}$. This angle participates in several trigonometric relationships but the easiest one to use is $\sec\theta = \frac{s}{y}$. Differentiating each side of this:

$$\begin{aligned}
\frac{d}{dt}\sec\theta &= \frac{d}{dt}\frac{s}{0.6} \\
\frac{d\theta}{dt}\frac{d}{d\theta}\sec\theta &= \frac{\frac{ds}{dt}}{0.6} \\
\frac{d\theta}{dt}\sec\theta\tan\theta &= \frac{-160}{0.6} \\
\frac{d\theta}{dt}\frac{s}{y}\frac{x}{y} &= \frac{-160}{0.6} \\
\frac{d\theta}{dt} &= \frac{-160y^2}{0.6xs} = \frac{-160(0.6)^2}{0.6\cdot 0.8\cdot 1} = -120
\end{aligned}$$

so you swivel your radargun towards due north (i.e. clockwise) at 120 radians per hour. For comparison, this would be a little under 2 degrees per second.

3. **(5 points)** *Use linear (a.k.a. differential) approximations to determine good rational estimates for the following values.*

   (a) $\sqrt{35.91}$.
   $f(a+h) \approx f(a) + hf'(a) = 6 + (-0.09)\frac{1}{12} = 5.9925$. The actual value of $\sqrt{35.91}$, for reference, is approximately 5.9924953.

   (b) $(-3.02)^4$.
   $f(a+h) \approx f(a) + hf'(a) = 81 + (-0.02)(-108) = 83.16$. The actual value of $(-3.02)^4$, for reference, is 83.18169616.