

This test is closed-book and closed-notes. For full credit show all of your work (legibly!), unless otherwise specified.

1. **(5 points)** Find a value k such that the matrix $\begin{pmatrix} 1 & 2 & 4 \\ 5 & k & 0 \\ -3 & -2 & 1 \end{pmatrix}$ is singular.

2. **(10 points)** For each of the following subsets S of a named vector space V , explain whether S is or is not a subspace of V and why.

(a) $V = \mathbb{R}^{2 \times 2}$, $S = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a + d = 0 \right\}$.

(b) $V = \mathbb{R}^3$, $S = \{(x, 3x, 4x + 2)^T : x \in \mathbb{R}\}$.

(c) $V = P_2$, $S = \{f(x) : f(2) \leq 0\}$.

3. **(10 points)** Let $A = \begin{pmatrix} 4 & 1 & 6 \\ 2 & 3 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 3 & 0 \\ -2 & 2 & -4 \end{pmatrix}$, and $\mathbf{v} = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$. For each of the following arithmetic expressions, either calculate its value or explain briefly why it cannot be calculated.

(a) $(A + B)\mathbf{v}$.

(b) AB .

(c) AB^T .

(d) $A^T B\mathbf{v}$.

4. **(15 points)** Let $A = \begin{pmatrix} 1 & 4 & 3 \\ -1 & -2 & 0 \\ 2 & 2 & 3 \end{pmatrix}$.

(a) **(10 points)** Calculate the inverse of A .

(b) **(5 points)** Find a solution \mathbf{v} to the matrix equation

$$\begin{pmatrix} 1 & 4 & 3 \\ -1 & -2 & 0 \\ 2 & 2 & 3 \end{pmatrix} \mathbf{v} = \begin{pmatrix} 12 \\ -12 \\ 8 \end{pmatrix}$$

5. (25 points) Answer the following questions.

(a) (10 points) Calculate the determinant $\begin{vmatrix} 1 & 1 & 1 & 3 \\ 0 & 3 & 1 & 3 \\ 0 & 0 & 2 & 2 \\ -1 & -1 & -1 & 2 \end{vmatrix}$.

(b) (7 points) What is the entry in the first row and second column of $\begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & 3 & 1 & 3 \\ 0 & 0 & 2 & 2 \\ -1 & -1 & -1 & 2 \end{pmatrix}^{-1}$?

(c) (8 points) Calculate the value of y in the following equation:

$$\begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & 3 & 1 & 3 \\ 0 & 0 & 2 & 2 \\ -1 & -1 & -1 & 2 \end{pmatrix} \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 0 \\ 2 \end{pmatrix}$$

6. **(35 points)** Answer the following related questions:

- (a) **(15 points)** For the following system of equations, determine its solution set or describe it as inconsistent:

$$\begin{cases} x_1 - x_2 + 3x_3 + 2x_4 = 1 \\ -x_1 + x_2 - 2x_3 + x_4 = -2 \\ 2x_1 - 2x_2 + 7x_3 + 7x_4 = 1 \end{cases}$$

- (b) **(10 points)** Calculate the dimension of the nullspace of the matrix $\begin{pmatrix} 1 & -1 & 3 & 2 \\ -1 & 1 & -2 & 1 \\ 2 & -2 & 7 & 7 \end{pmatrix}$, and find a basis.

- (c) **(5 points)** Is the set of vectors $\left\{ \begin{pmatrix} 1 \\ -1 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \\ 7 \\ 7 \end{pmatrix} \right\}$ linearly independent? Explain your reasoning.

- (d) **(5 points)** Is the set of vectors $\left\{ \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 3 \\ -2 \\ 7 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 7 \end{pmatrix} \right\}$ linearly independent? Explain your reasoning.