This test is closed-book and closed-notes. For full credit show all of your work (legibly!), unless otherwise specified.

1. **(15 points)** The matrix $A = \begin{pmatrix} 2 & 2 & 4 & 7 \\ 1 & 6 & 12 & 6 \\ -2 & 2 & 4 & -5 \end{pmatrix}$ is row-equivalent to $\begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 2 & 4 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$. Using this information, determine a basis for each of the following spaces.

   (a) **(5 points)** $N(A)$.

   (b) **(5 points)** $R(A)$.

   (c) **(5 points)** $R(A^T)$.

2. **(15 points)** Find a least-squares solution to the following inconsistent system of equations:

$$\begin{cases} x + y = 6 \\ x - y = 1 \\ \quad 2y = 9 \end{cases}$$

3. **(15 points)** For each of the following maps L on a vector space V, determine with explanation whether L is a linear transformation.

   (a) $L(f) = e^{f(x)}$ on $V = C(\mathbb{R})$, the space of continuous functions on the real number line.

   (b) $L((x, y)^{\mathrm{T}}) = (x^2 - y^2, x^2 + y^2)^{\mathrm{T}}$ on $V = \mathbb{R}^2$.

   (c) $L(f) = (x + 1)f''(x)$ on $V = P_3$.

4. **(15 points)** Given the inner product $\langle f, g \rangle = f(1)g(1) + f(0)g(0)$ on the vector space P_2, answer the following questions.

   (a) **(5 points)** Determine the projection of x^2 onto $2x - 1$.

   (b) **(10 points)** Find a nonzero vector which is orthogonal to $x + 3$.

5. **(10 points)** Answer the following questions.

   (a) **(5 points)** What is the distance from the point $(1, 6)$ to the line $y = -2x + 5$?

   (b) **(5 points)** What is the distance from $(0, 1, 1)$ to the plane $4x - 5y + z = 0$?

6. **(15 points)** Answer the following questions about the linear transformation L on $\mathbf{R}^3$ here described: $L((x, y, z)^{\mathrm{T}}) = (2x - y, y + z, x + 2z)^{\mathrm{T}}$.

   (a) **(5 points)** What matrix represents L with respect to the standard basis $[\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3]$?

   (b) **(5 points)** For $\mathbf{u}_1 = (0, 1, 1)^{\mathrm{T}}$, $\mathbf{u}_2 = (2, 1, 0)^{\mathrm{T}}$, and $\mathbf{u}_3 = (1, 1, 0)^{\mathrm{T}}$, what matrix represents L with respect to the nonstandard basis $[\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3]$?

   (c) **(5 points)** Determine a basis for the kernel of L.

7. **(15 points)** Use the Gram-Schmidt process to orthonormalize the basis $\{(3, 4, 0)^{\mathrm{T}}, (1, 2, -1)^{\mathrm{T}}, (0, 1, 1)^{\mathrm{T}}\}$.