

This test is closed-book and closed-notes. For full credit show all of your work (legibly!), unless otherwise specified.

1. **(15 points)** For the following system of equations, determine its solution set or describe it as inconsistent:

$$\begin{cases} x_1 - 3x_2 - 6x_3 - 5x_4 = -13 \\ 2x_1 + x_2 + 2x_3 + 11x_4 = 9 \\ 3x_1 + x_2 + 2x_3 + 15x_4 = 11 \end{cases}$$

2. **(10 points)** Let  $A = \begin{pmatrix} 2 & 0 & -4 \\ 3 & 1 & 0 \end{pmatrix}$ ,  $B = (1 \ -1)$ , and  $\mathbf{v} = \begin{pmatrix} 5 \\ 2 \\ 0 \end{pmatrix}$ . For each of the following arithmetic expressions, either calculate its value or explain briefly why it cannot be calculated.

(a)  $B\mathbf{v}$ .

(b)  $A\mathbf{v}$ .

(c)  $2A + (\mathbf{v}B)^T$ .

(d)  $BA$ .

3. **(20 points)** Let  $A = \begin{pmatrix} 2 & 1 & 2 \\ 3 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}$ .

(a) **(12 points)** Calculate the inverse of  $A$ .

(b) **(5 points)** Find a solution  $\mathbf{y}$  to the matrix equation

$$\begin{pmatrix} 2 & 1 & 2 \\ 3 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix} \mathbf{y} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$$

(c) **(3 points)** Calculate the dimension of the nullspace of  $A$ , and determine a basis for it.

4. **(5 points)** Find a value  $k$  such that the matrix  $\begin{pmatrix} 2 & 1 & 0 \\ 3 & -5 & k \\ -1 & 2 & 1 \end{pmatrix}$  is singular.

5. (25 points) Answer the following questions.

(a) (10 points) Calculate the determinant  $\begin{vmatrix} 1 & -3 & 2 & 5 \\ 1 & -3 & 2 & 4 \\ 0 & 1 & -3 & 2 \\ 2 & -1 & 0 & 3 \end{vmatrix}$ .

(b) (8 points) Calculate the value of  $x$  (and only  $x$ ) in the following equation:

$$\begin{pmatrix} 1 & -3 & 2 & 5 \\ 1 & -3 & 2 & 4 \\ 0 & 1 & -3 & 2 \\ 2 & -1 & 0 & 3 \end{pmatrix} \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 0 \\ 1 \end{pmatrix}$$

6. (7 points) What is the entry in the second row and third column of  $\begin{pmatrix} 1 & -3 & 2 & 5 \\ 1 & -3 & 2 & 4 \\ 0 & 1 & -3 & 2 \\ 2 & -1 & 0 & 3 \end{pmatrix}^{-1}$  ?

7. **(10 points)** For each of the following subsets  $S$  of a named vector space  $V$ , explain whether  $S$  is or is not a subspace of  $V$  and why.

(a)  $V = \mathbb{R}^{3 \times 3}$ ,  $S = \{A : \det A = 0\}$ .

(b)  $V = \mathbb{R}^4$ ,  $S = \{(w, x, y, z)^T : w + x + y + z = 0\}$ .

(c)  $V = P_5$ ,  $S = \{f(x) : \text{the degree of } f \text{ is less than } 3\}$ .

8. **(15 points)** Answer the following questions related to the matrix  $A = \begin{pmatrix} 1 & 2 & -3 & -1 \\ -2 & -1 & 5 & 2 \\ -1 & 4 & 1 & 1 \end{pmatrix}$ .

(a) **(10 points)** Calculate the dimension of the nullspace of  $A$ , and find a basis.

(b) **(10 points)** Is the set of vectors  $\left\{ \begin{pmatrix} 1 \\ 2 \\ -3 \\ -1 \end{pmatrix}, \begin{pmatrix} -2 \\ -1 \\ 5 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ 4 \\ 1 \\ 1 \end{pmatrix} \right\}$  linearly independent?

Explain your reasoning.