This test is closed-book and closed-notes. For full credit show all of your work (legibly!), unless otherwise specified.

1. **(15 points)** For the matrix $A = \begin{pmatrix} -3 & 1 & 3 & 4 \\ 1 & 2 & -1 & -2 \\ -3 & 9 & 4 & 2 \end{pmatrix}$, determine a basis for each of the following spaces.

   (a) **(5 points)** $N(A)$.

   (b) **(5 points)** $R(A)$.

   (c) **(5 points)** $R(A^T)$.

2. **(15 points)** Find a least-squares solution to the following inconsistent system of equations:

$$\begin{cases} 2x + 3y = 6 \\ -x - 3y = -2 \\ x = 3 \end{cases}$$

3. **(15 points)** For each of the following maps L on a vector space V, determine with explanation whether L is a linear transformation.

   (a) $L(f) = \int_0^x f(t)dt$ on $V = C[0,1]$, the space of continuous functions on $[0,1]$.

   (b) $L((x,y)^{\mathrm{T}}) = (3x,1)^{\mathrm{T}}$ on $V = \mathbb{R}^2$.

   (c) $L(f) = f(1)x + f(0)$ on $V = P_3$.

4. **(15 points)** Given the inner product $\langle f, g\rangle = \int_0^1 f(x)g(x)dx$ on the vector space P_2, answer the following questions.

   (a) **(5 points)** Determine the cosine of the angle between the vectors $1 + x$ and $x^2 - 2$.

   (b) **(10 points)** Find a nonzero vector which is orthogonal to $x^2 + 2x$.

5. **(10 points)** Answer the following questions.

   (a) **(5 points)** What is the distance from the point $(2,1)$ to the line $y = 4x$?

   (b) **(5 points)** What is the nearest point to $(1,2,3)$ on the line between $(0,0,0)$ and $(-2,1,4)$?

6. **(15 points)** Answer the following questions about the linear transformation L on $\mathbf{R}^3$ here described: $L((x, y, z)^{\mathrm{T}}) = (x + 2z, 2x + 4z, y)^{\mathrm{T}}$.

   (a) **(5 points)** What matrix represents L with respect to the standard basis $[\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3]$?

   (b) **(5 points)** For $\mathbf{u}_1 = (1, 1, 1)^{\mathrm{T}}$, $\mathbf{u}_2 = (1, 0, 1)^{\mathrm{T}}$, and $\mathbf{u}_3 = (1, -1, 0)^{\mathrm{T}}$, what matrix represents L with respect to the nonstandard basis $[\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3]$?

   (c) **(5 points)** Determine a basis for the kernel of L.

7. **(15 points)** Use the Gram-Schmidt process to orthonormalize the basis $\{(1, 2, 2)^{\mathrm{T}}, (1, 1, 0)^{\mathrm{T}}, (0, 1, 1)^{\mathrm{T}}\}$.