1. **(12 points)** *Work through the following process for the following system of equations:*

$$\begin{cases} x_1 + 3x_2 + x_3 + x_4 = 3 \\ 2x_1 - 2x_2 - 6x_3 + 2x_4 = 2 \\ x_1 + 2x_2 + x_4 = 9 \end{cases}$$

(a) **(1 point)** *Write an augmented matrix for this system.*

$$\left[\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 3 \\ 2 & -2 & -6 & 2 & 2 \\ 1 & 2 & 0 & 1 & 9 \end{array}\right]$$

(b) **(8 point)** *Convert the augmented matrix above to reduced row-echelon form.*

$$\left[\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 3 \\ 2 & -2 & -6 & 2 & 2 \\ 1 & 2 & 0 & 1 & 9 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 3 \\ 0 & -8 & -8 & 0 & -4 \\ 1 & 2 & 0 & 1 & 9 \end{array}\right]$$
$$\to \left[\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 3 \\ 0 & -8 & -8 & 0 & -4 \\ 0 & -1 & -1 & 1 & 6 \end{array}\right]$$
$$\to \left[\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 3 \\ 0 & 1 & 1 & 0 & \frac{1}{2} \\ 0 & -1 & -1 & 0 & 6 \end{array}\right]$$
$$\to \left[\begin{array}{cccc|c} 1 & 0 & -2 & 1 & \frac{3}{2} \\ 0 & 1 & 1 & 0 & \frac{1}{2} \\ 0 & -1 & -1 & 2 & 6 \end{array}\right]$$
$$\to \left[\begin{array}{cccc|c} 1 & 0 & -2 & 1 & \frac{3}{2} \\ 0 & 1 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{13}{2} \end{array}\right]$$
$$\to \left[\begin{array}{cccc|c} 1 & 0 & -2 & 1 & \frac{3}{2} \\ 0 & 1 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

(c) **(3 point)** *Indicate whether the system is inconsistent, has a unique solution, or has multiple solutions. If the system is consistent, identify its solution or solutions.*

Since the constant row is a pivot, the system is inconsistent.

2. **(8 points)** *For the two matrices given below, either calculate each of the following expressions, or briefly explain why it cannot be calculated.*

$$A = \begin{bmatrix} 5 & 2 & 0 \\ -3 & 1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} -4 & -1 \\ 0 & 3 \end{bmatrix}$$

(a) **(1 points)** $A - B$.

Because A and B have different dimensions, they cannot be subtracted.

(b) **(1 points)** $4B$.

$$4\begin{bmatrix} -4 & -1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 4\cdot -4 & 4\cdot -1 \\ 4\cdot 0 & 4\cdot 3 \end{bmatrix} = \begin{bmatrix} -16 & -4 \\ 0 & 12 \end{bmatrix}.$$

(c) **(2 points)** A^{T}.

$$\begin{bmatrix} 5 & 2 & 0 \\ -3 & 1 & 1 \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 5 & -3 \\ 2 & 1 \\ 0 & 1 \end{bmatrix}.$$

(d) **(2 points)** AB.

Since A has 3 columns and B has 2 rows, they cannot be multiplied (in this order).

(e) **(2 points)** BA.

$$\begin{bmatrix} -4 & -1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 5 & 2 & 0 \\ -3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -4\cdot 5 - 1\cdot -3 & -4\cdot 2 - 1\cdot 1 & -4\cdot 0 - 1\cdot 1 \\ 0\cdot 5 + 3\cdot -3 & 0\cdot 2 + 3\cdot 1 & 0\cdot 0 + 3\cdot 1 \end{bmatrix} = \begin{bmatrix} -17 & -9 & -1 \\ -9 & 3 & 3 \end{bmatrix}$$